Fundamentals of Electronic Circuit Design

By
Hongshen Ma

Preface – Why Study Electronics?

Purely mechanical problems are often only a subset of larger multi-domain problems faced by the designer. Particularly, the solutions of many of today's interesting problems require expertise in both mechanical engineering and electrical engineering. DVD players, digital projectors, modern cars, machine tools, and digital cameras are just a few examples of the results of such combined innovation. In these hybrid systems, design trade-offs often span the knowledge space of both mechanical and electrical engineering. For example, in a car engine, is it more cost-effective to design a precise mechanical timing mechanism to trigger the firing of each cylinder, or is it better to use electronic sensors to measure the positions of each piston and then use a microprocessor to trigger the firing? For every problem, designers with combined expertise in mechanical and electrical engineering will be able to devise more ideas of possible solutions and be able to better evaluate the feasibility of each idea.

A basic understanding of electronic circuits is important even if the designer does not intend to become a proficient electrical engineer. In many real-life engineering projects, it is often necessary to communicate, and also negotiate, specifications between engineering teams having different areas of expertise. Therefore, a basic understanding of electronic circuits will allow the mechanical engineer to evaluate whether or not a given electrical specification is reasonable and feasible.

The following text is designed to provide an efficient introduction to electronic circuit design. The text is divided into two parts. Part I is a barebones introduction to basic electronic theory while Part II is designed to be a practical manual for designing and building working electronic circuits.

Part I

Fundamentals Principles

By
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Important note:

This document is a rough draft of the proposed textbook. Many of the sections and figures need to be revised and/or are missing. Please check future releases for more complete versions of this text.

Fundamentals of Electronic Circuit Design

Outline

Part I – Fundamental Principles

1 The Basics

1.1 Voltage and Current

Voltage is the difference in electrical potential between two points in space. It is a measure of the amount of energy gained or lost by moving a unit of positive charge from one point to another, as shown in *Figure 1.1*. Voltage is measured in units of Joules per Coulomb, known as a Volt (V). It is important to remember that voltage is not an absolute quantity; rather, it is always considered as a relative value between two points. In an electronic circuit, the electromagnetic problem of voltages at arbitrary points in space is typically simplified to voltages between nodes of circuit components such as resistors, capacitors, and transistors.

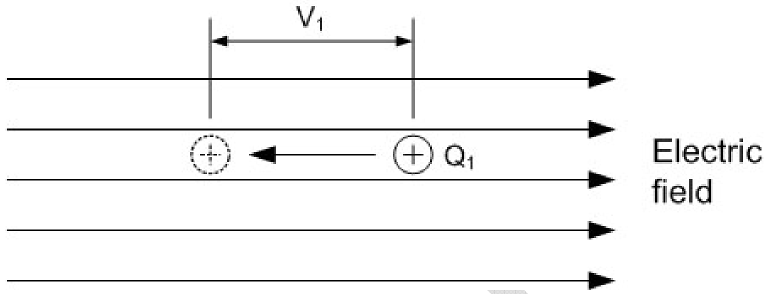

Figure 1.1: Voltage V_1 is the electrical potential gained by moving charge Q_1 in an electric field.

When multiple components are connected in parallel, the voltage drop is the same across all components. When multiple components are connected in series, the total voltage is the sum of the voltages across each component. These two statements can be generalized as Kirchoff's Voltage Law (KVL), which states that the sum of voltages around any closed loop (e.g. starting at one node, and ending at the same node) is zero, as shown in *Figure 1.2*.

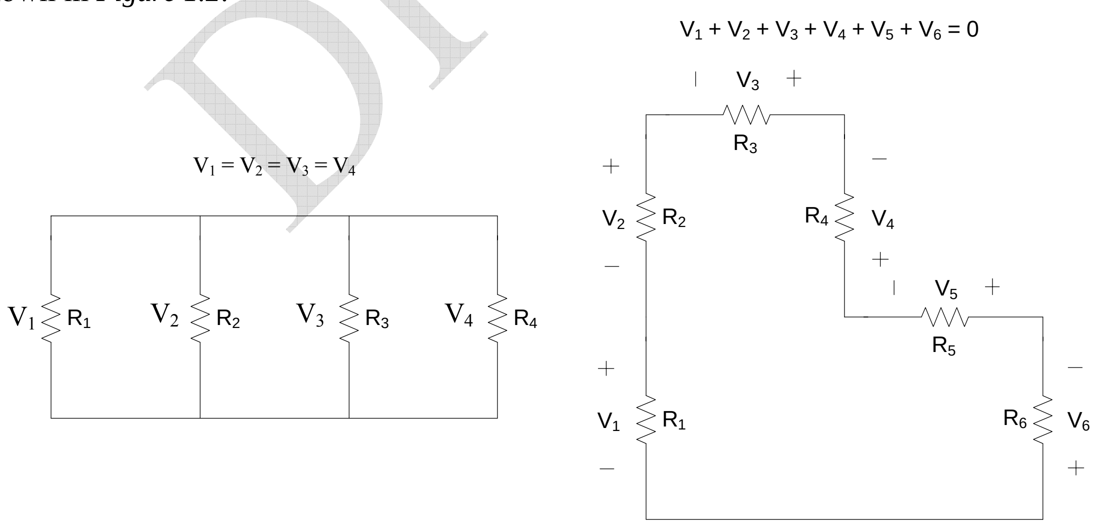

Figure 1.2: Kirchoff's Voltage Law: The sum of the voltages around any loop is zero.

Electric current is the rate at which electric charge flows through a given area. Current is measured in the unit of Coulombs per second, which is known as an ampere

(A). In an electronic circuit, the electromagnetic problem of currents is typically simplified as a current flowing through particular circuit components.

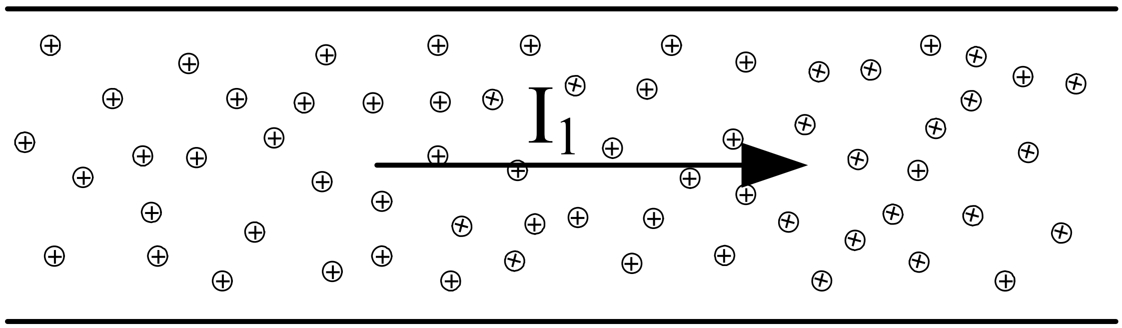

Figure 1.3: Current I_1 is the rate of flow of electric charge

When multiple components are connected in series, each component must carry the same current. When multiple components are connected in parallel, the total current is the sum of the currents flowing through each individual component. These statements are generalized as Kirchoff's Current Law (KCL), which states that the sum of currents entering and exiting a node must be zero, as shown in *Figure 1.4*.

$$I_1 + I_2 + I_3 + I_4 = 0$$

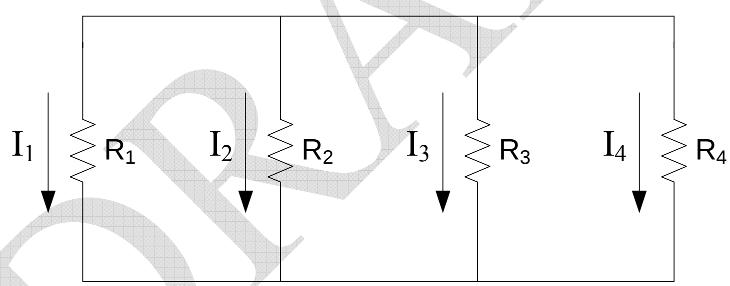

Figure 1.4: Kirchoff's Current Law – the sum of the currents going into a node is zero.

An intuitive way to understand the behavior of voltage and current in electronic circuits is to use hydrodynamic systems as an analogue. In this system, voltage is represented by gravitational potential or height of the fluid column, and current is represented by the fluid flow rate. Diagrams of these concepts are show in *Figure 1.5* through *1.7*. As the following sections will explain, electrical components such as resistors, capacitors, inductors, and transistors can all be represented by equivalent mechanical devices that support this analogy.

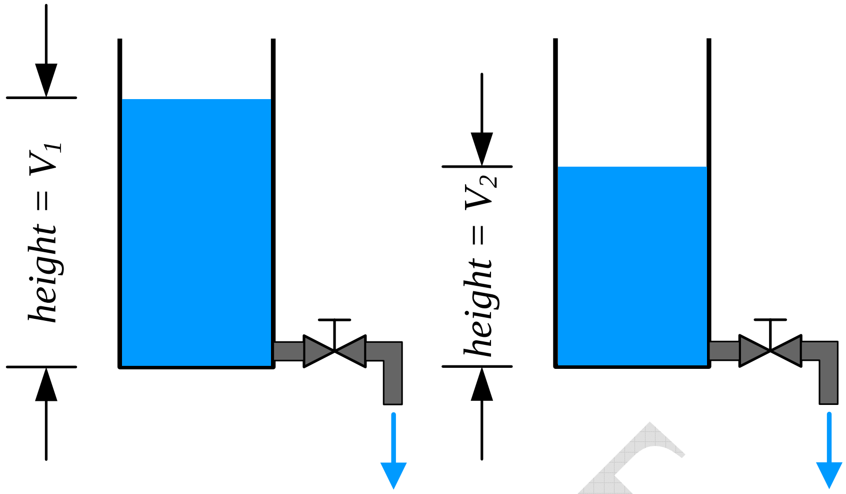

Figure 1.5: Hydrodynamic analogy for voltage

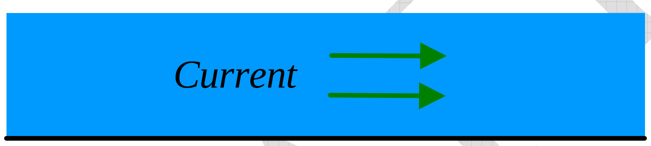

Figure 1.6: Hydrodynamic analogy for current

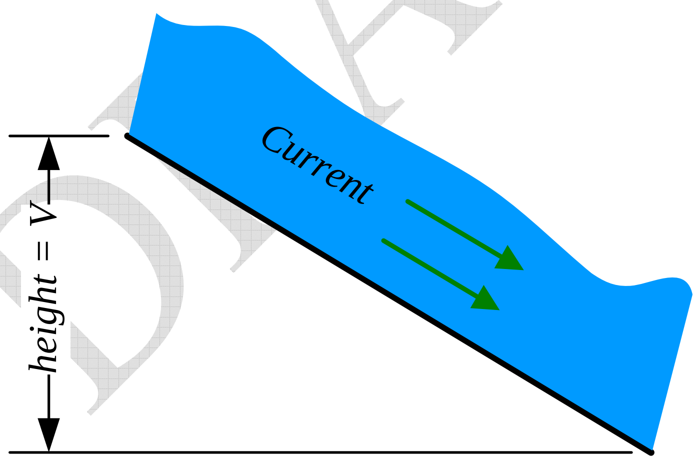

Figure 1.7: A hydrodynamic example representing both voltage and current

1.2 Resistance and Power

When a voltage is applied across a conductor, a current will begin to flow. The ratio between voltage and current is known as resistance. For most metallic conductors, the relationship between voltage and current is linear. Stated mathematically, this property is known as Ohm's law, where

$$R = \frac{V}{I}$$

Some electronic components such diodes and transistors do not obey Ohm's law and have a non-linear current-voltage relationship.

The power dissipated by a given circuit component is the product of voltage and current,

$$P = IV$$

The unit of power is the Joule per second (*J/s*), which is also known as a Watt (*W*). If a component obeys Ohm's law, the power it dissipates can be equivalently expressed as,

$$P = I^2 R \text{ or}$$

$$P = \frac{V^2}{R}.$$

1.3 Voltage and Current Sources

There are two kinds of energy sources in electronic circuits: voltage sources and current sources. When connected to an electronic circuit, an ideal voltage source maintains a given voltage between its two terminals by providing any amount of current necessary to do so. Similarly, an ideal current source maintains a given current to a circuit by providing any amount of voltage across its terminals necessary to do so.

Voltage and current sources can be independent or dependent. Their respective circuit symbols are shown in *Figure 1.8*. Independent sources are usually shown as a circle while dependent sources are usually shown as a diamond-shape. Independent sources can have a DC output or a functional output; some examples are a sine wave, square wave, impulse, and linear ramp. Dependent sources can be used to implement a voltage or current which is a function of some other voltage or current in the circuit. Dependent sources are often used to model active circuits that are used for signal amplification.

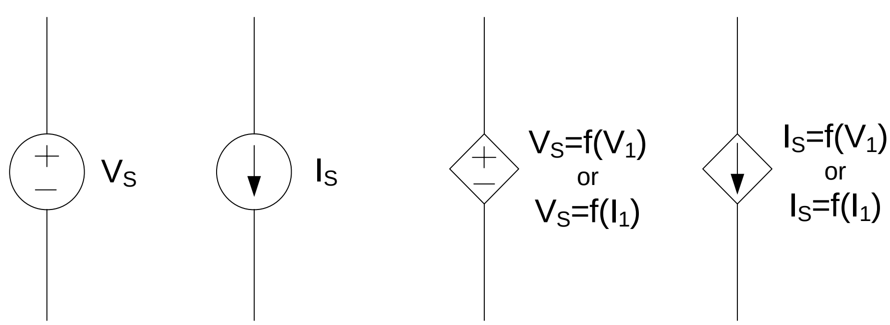

Figure 1.8: Circuit symbols for independent and dependent voltage and current sources

1.4 Ground

An often used and sometimes confusing term in electronic circuits is the word *ground*. The ground is a circuit node to which all voltages in a circuit are referenced. In a constant voltage supply circuit, one terminal from each voltage supply is typically connected to ground, or is *grounded*. For example, the negative terminal of a positive power supply is usually connected to ground so that any current drawn out of the positive terminal can be put back into the negative terminal via ground. Some circuit symbols used for ground are shown in *Figure 1.9*.

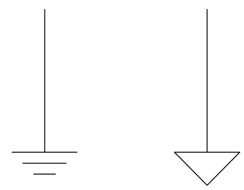

Figure 1.9: Circuit symbols used for ground

In some circuits, there are *virtual grounds*, which are nodes at the same voltage as ground, but are not connected to a power supply. When current flows into the virtual ground, the voltage at the virtual ground may change relative to the real ground, and the consequences of this situation must be analyzed carefully.

1.5 Electronic Signals

Electronic signals are represented either by voltage or current. The time-dependent characteristics of voltage or current signals can take a number of forms including DC, sinusoidal (also known as AC), square wave, linear ramps, and pulse-width modulated signals. Sinusoidal signals are perhaps the most important signal forms since once the circuit response to sinusoidal signals are known, the result can be generalized to predict how the circuit will respond to a much greater variety of signals using the mathematical tools of Fourier and Laplace transforms.

A sinusoidal signal is specified by its amplitude (A), angular frequency (ω), and phase (ϕ) as,

$$V(t) = A\sin\left(\omega t + \phi\right)$$

When working with sinusoidal signals, the mathematical manipulations often involves computing the effects of the circuit on the amplitude and phase of the signal, which can involve cumbersome trigonometric identities. Operations involving sinusoidal functions can be greatly simplified using the mathematical construct of the complex domain (see Appendix for more information). The sinusoidal signal from the above equation, when expressed in the complex domain, becomes the complex exponential,

$$V(t) = Ae^{-j\omega t},$$

where the physical response is represented by the real part of this expression. The amplitude and phase of the signal are both described by the complex constant *A*, where

$$A = |A| e^{-j\phi_A}.$$

As the following section will show, the complex representation of electronic signals greatly simplifies the analysis of electronic circuits.

1.6 Electronic Circuits as Linear Systems

Most electronic circuits can be represented as a system with an input and an output as shown in *Figure 1.10*. The input signal is typically a voltage that is generated by a sensor or by another circuit. The output signal is also often a voltage and is used to power an actuator or transmit signals to another circuit.

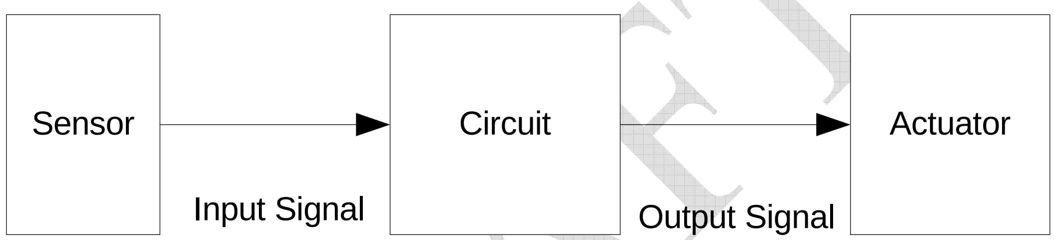

Figure 1.10: Electronic circuit represented as a linear system

In many instances, it is possible to model the circuit as a linear system, which can be described by the transfer function H, such that

$$H = \frac{V_{out}}{V_{in}}.$$

For DC signals, the linearity of the system implies that H is independent of V_{in}. For dynamic signals, the transfer function cannot in general be described simply. However, if the input is a sinusoidal signal then the output must also be a sinusoidal signal with the same frequency but possibly a different amplitude and phase. In other words, **a linear system can only modify the amplitude and phase of a sinusoidal input.** As a result, if the input signal is described as a complex exponential,

$$V_{in}(t) = Ae^{-j\omega_o t},$$

where A is a complex constant,

$$A = |A| e^{-j\phi_A}.$$

The transfer function H can be entirely described by a complex constant,

$$H = |H| e^{-\mathrm{j}\phi_H},$$

and the output signal is simply

$$V_{out}(t) = HAe^{-\mathrm{j}\omega_o t},$$

or in expanded form,

$$V_{out}(t) = |H||A| e^{-\mathrm{j}(\phi_A + \phi_H)} e^{-\mathrm{j}\omega_o t}.$$

In general, the sinusoidal response of linear systems is not constant over frequency and H is also a complex function of ω.

2 Fundamental Components: Resistors, Capacitors, and Inductors

Resistors, capacitors, and inductors are the fundamental components of electronic circuits. In fact, all electronic circuits can be equivalently represented by circuits of these three components together with voltage and current sources.

2.1 Resistors

Resistors are the most simple and most commonly used electronic component. Resistors have a linear current-voltage relationship as stated by Ohm's law. The unit of resistance is an *ohm*, which is represented by the letter omega (Ω). Common resistor values range from 1 Ω to 22 MΩ.

In the hydrodynamic analogy of electronic circuits, resistors are equivalent to a pipe. As fluid flows through a pipe, frictional drag forces at the walls dissipate energy from the flow and thus reducing the pressure, or equivalently, the potential energy of the fluid in the pipe. A small resistor is equivalent to a large diameter pipe that will allow for a high flow rate, whereas a large resistor is equivalent to a small diameter pipe that greatly constricts the flow rate, as shown in *Figure 2.1*.

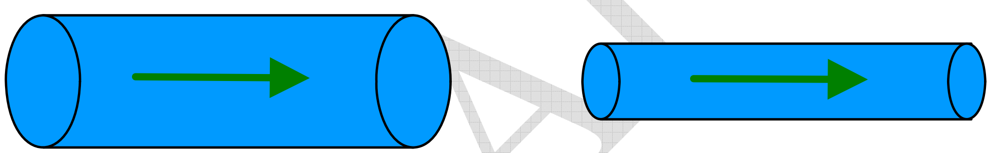

Figure 2.1: The hydrodynamic model of a resistor is a pipe

When several resistors are connected in series, the equivalent resistance is the sum of all the resistances. For example, as shown in *Figure 2.2*,

$$R_{eq} = R_1 + R_2$$

When several resistors are connected in parallel, the equivalent resistance is the inverse of the sum of their inverses. For example,

$$R_{eq} = \frac{1}{\dfrac{1}{R_3} + \dfrac{1}{R_4}} = \frac{R_3 R_4}{R_3 + R_4}$$

In order to simplify this calculation when analyzing more complex networks, electrical engineers use the || symbol to indicate that two resistances are in parallel such that

$$R_3 \parallel R_4 = \frac{1}{\dfrac{1}{R_3} + \dfrac{1}{R_4}} = \frac{R_3 R_4}{R_3 + R_4}$$

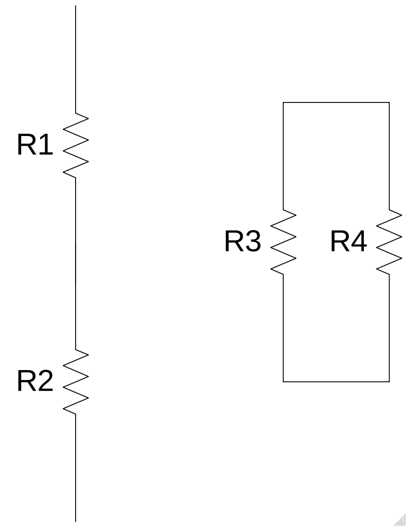

Figure 2.2: Resistors in series and in parallel

A common resistor circuit is the voltage divider used to divide a voltage by a fixed value. As shown in *Figure 2.3*, for a voltage V_{in} applied at the input, the resulting output voltage is

$$V_{out} = V_{in} \frac{R_1}{R_1 + R_2}$$

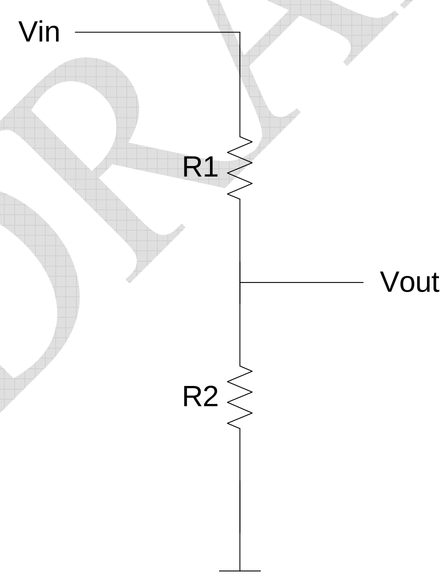

Figure 2.3: Voltage divider circuit

<u>2.2 Capacitors</u>

A capacitor is a device that stores energy in the form of voltage. The most common form of capacitors is made of two parallel plates separated by a dielectric material. Charges of opposite polarity can be deposited on the plates, resulting in a voltage V across the capacitor plates. Capacitance is a measure of the amount of electrical charge required to build up one unit of voltage across the plates. Stated mathematically,

$$C = \frac{Q}{V_C},$$

where Q is the number of opposing charge pairs on the capacitor. The unit of capacitance is the Farad (F) and capacitors are commonly found ranging from a few picofarads (pF) to hundreds of microfarads (μF).

In the hydrodynamic analogy to electronic circuits, a capacitor is equivalent to a bottle, as shown in *Figure 2.4*. The voltage across the capacitor is represented by the height of fluid in the bottle. As fluid is added to the bottle, the fluid level rises just as charges flowing onto the capacitor plate build up the voltage. A small capacitor is a thin bottle, where adding a small volume of fluid quickly raise the fluid level. Correspondingly, a large capacitor is a wide bottle, where a larger volume of fluid is required to raise the fluid level by the same distance.

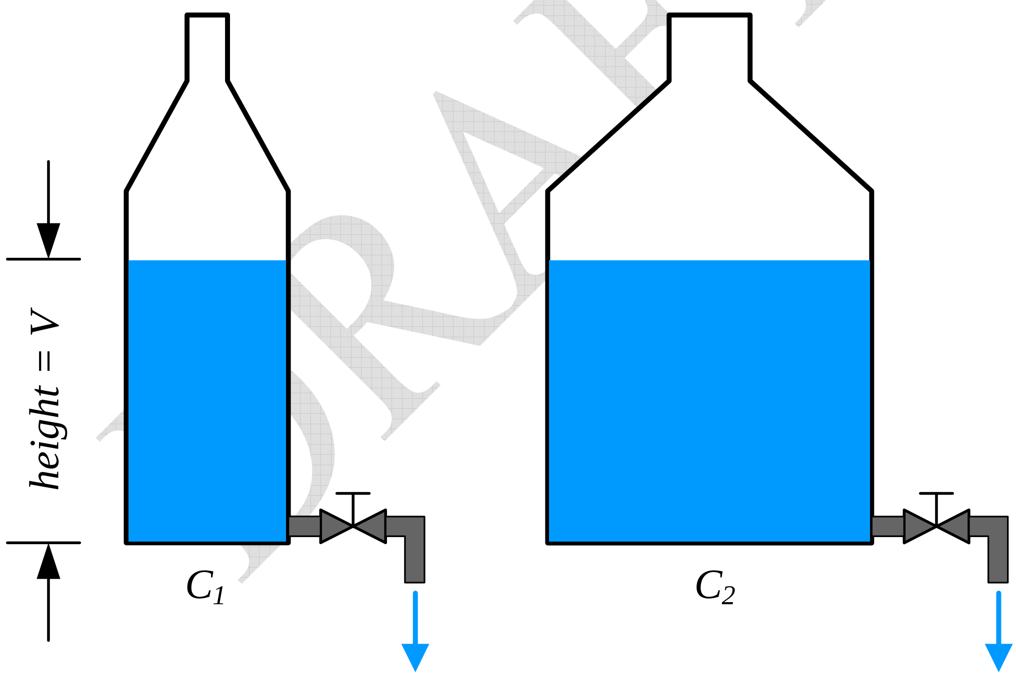

Figure 2.4: The hydrodynamic model of a capacitor is a bottle

The current-voltage relationship of the capacitor is obtained by differentiating $Q = CV$ to get

$$I = \frac{dQ}{dt} = C\frac{dV_C}{dt}.$$

Unlike a resistor, current in a capacitor is proportional to the derivative of voltage rather than voltage itself. Alternatively, it can be said that the voltage on a capacitor is proportional to the time integral of the influx current.

$$V_C = \int I dt$$

A typical example of a capacitor circuit is shown in *Figure 2.5*, where the capacitor is connected in series with a resistor, a switch, and an ideal voltage source. Initially, for $t < 0$, the switch is open and the voltage on the capacitor is zero. The switch closes at $t = 0$, the voltage drop across the resistor is $V_{in} - V_C$, and charges flows onto the capacitor at the rate of $(V_{in} - V_C)/R$. As voltage builds on the capacitor, the corresponding voltage on the resistor is therefore decreased. The reduction in voltage leads to a reduction of the current through the circuit loop and slows the charging process. The exact behavior of voltage across the capacitor can be found by solving the first order differential equation,

$$C\frac{dV_C}{dt} = \frac{V_S - V_C}{R}.$$

The voltage across the capacitor behaves as an exponential function of time, which is shown in Figure ###. The term *RC*, is known as the time constant of the exponential function, and is often simply denoted as τ.

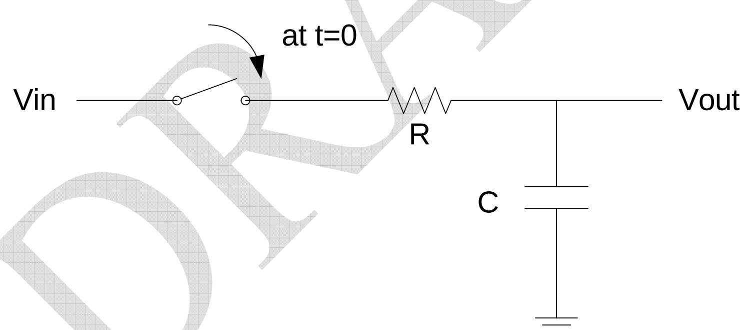

Figure 2.5: Simple RC circuit

(### insert plot of RC response here ###)

Figure ###: Voltage output for the RC circuit as a function of time

2.3 Inductor

An inductor is a device that stores energy in the form of current. The most common form of inductors is a wire wound into a coil. The magnetic field generated by the wire creates a counter-acting electric field which impedes changes to the current. This effect is known as Lenz's law and is stated mathematically as

$$V_L = -L\frac{dI}{dt}.$$

The unit of inductance is a Henry (H) and common inductors range from nanohenries (nH) to microhenries (μH).

In the hydrodynamic analogy of electronic circuits, an inductor can be thought of as a fluid channel pushing a flywheel as shown in *Figure 2.6*. When the fluid velocity (current) in the channel changes, the inertia of the flywheel tends to resist that change and maintain its original angular momentum. A large inductor corresponds to a flywheel with a large inertia, which will have a larger influence on the flow in the channel. Correspondingly a small inductor corresponds to a flywheel with a small inertia, which will have a lesser effect on the current.

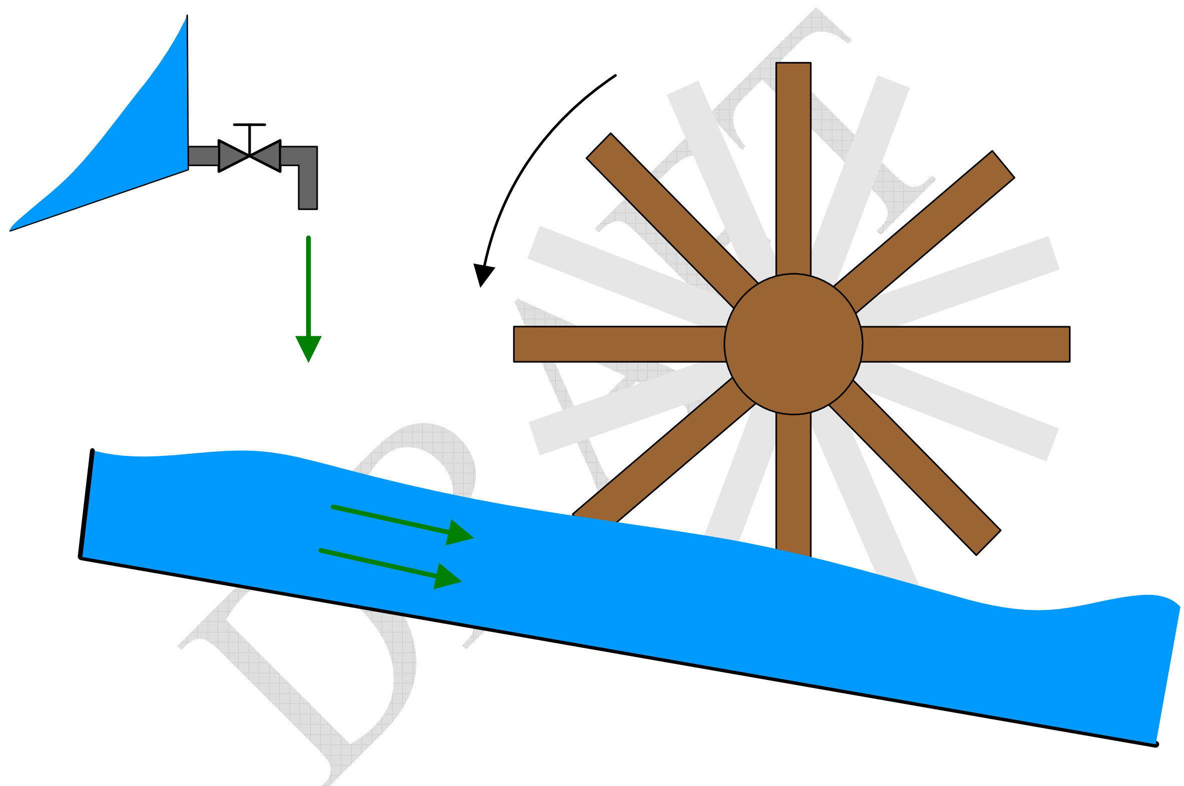

Figure 2.6: Hydrodynamic analogy of an inductor is a flywheel

An example of the time domain analysis of an inductor circuit is shown in *Figure 2.7* where the inductor is connected in series with a resistor, a switch, and an ideal voltage source. Initially, the current through the inductor is zero and the switch goes from closed to open at $t = 0$. Similar to the capacitor-resistor circuit, the time-domain behavior of this circuit can be determined by solving the first order differential equation ###. The resulting voltage across the inductor is an exponential function of time as shown in Figure ###.

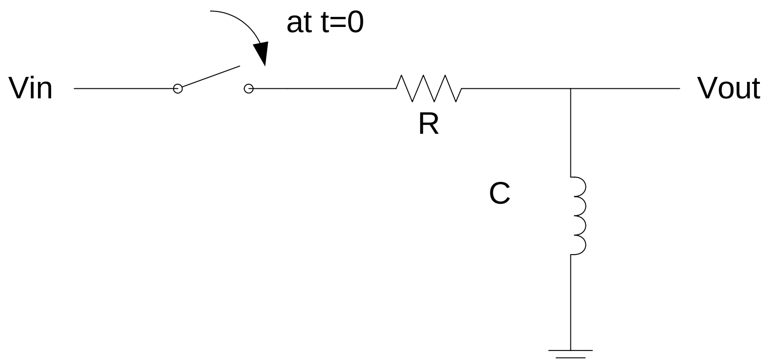

Figure 2.7: Simple RL circuit

###insert V(t) plot###
Figure ###: Voltage output for the RL circuit as a function of time

Switching circuits involving inductors have a rather destructive failure mode. Suppose that in the circuit shown in Figure ###, the switch is opened again after the current flowing through the inductor has reached steady-state. Since the current is terminated abruptly, the derivative term of Eq. ### can be very high. High voltages can result in electrical breakdown which can permanently damage the inductor as well as other components in the circuit. This problem and appropriate remedies are discussed in more detail in section 18.2.

3 Impedance and s-Domain Circuits

3.1 The Notion of Impedance

Impedance is one of the most important concepts in electronic circuits. The purpose of impedance is to generalize the idea of resistance to create a component, shown in *Figure 3.1*, to capture the behavior of resistors, capacitors, and inductors, for steady-state sinusoidal signals. This generalization is motivated by the fact that as long as the circuit is linear, its behavior can be analyzed using KVL and KCL.

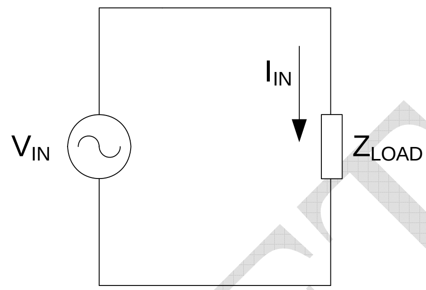

Figure 3.1: Impedance – a generalized component

Impedance essentially can be viewed as frequency-dependent resistance. While resistance of a circuit is the instantaneous ratio between voltage and current, impedance of a circuit is the ratio between voltage and current for steady-state sinusoidal signals, which can vary with of frequency. As the later parts of this section will show, the voltage and current caused by applying a steady-state sinusoidal signal to any combination of resistors, capacitors, and inductors, are related by a constant factor and a phase shift. Therefore, impedance can be expressed by a complex constant using an extended version Ohm's law,

$$Z(\omega) = \frac{V(\omega)}{I(\omega)},$$

Where $V(\omega)$ and $I(\omega)$ are both the complex exponential representations of sinusoidal functions as disused in the next section.

The real part and imaginary part of impedance are interpreted as a resistive part that dissipates energy and a reactive part that stores energy. Resistors can only dissipate energy and therefore their impedances have only a real part. Capacitors and inductors can only store energy and therefore their impedances have only an imaginary part. When resistors, capacitors, and inductors are combined, the overall impedance may have both real and imaginary parts. It is important to note that the definition of impedance preserves the definition of resistance. Therefore, for a circuit with only resistors, $Z_{EQ} = R_{EQ}$.

###An intuitive mechanical analog of impedance is stiffness. Stiffness is defined as the ratio between stress and strain, which in a practical mechanical structure can be measured as force and deflection. ###

3.2 The Impedance of a Capacitor

The impedance of a capacitor is determined by assuming that a sinusoidal voltage is applied across the capacitor, such that

$$V_C = Ae^{j\omega t}.$$

Since

$$I_C = C\frac{dV_C}{dt},$$

the resulting current is

$$I_C = j\omega CAe^{j\omega t}.$$

Therefore, the impedance, or the ratio between voltage and current, is

$$Z_C = \frac{V_C}{I_C} = \frac{1}{j\omega C}.$$

From the above equation, it is possible to see that the impedance of a capacitor is a frequency-dependent resistance that is inversely proportional to frequency; Z_C is small at high frequency and large at low frequency. At DC, the impedance of a capacitor is infinite. The impedance expression also indicates that for a sinusoidal input, the current in a capacitor lags its voltage by a phase of 90^o.

3.3 Simple RC Filters

A simple low-pass filter circuit, which allows low frequency signals to pass through the circuit while attenuating high-frequency signals, can be made using a resistor and capacitor in series as shown in *Figure 3.2*. The transfer function of this filter can be determined by analyzing the circuit as a voltage divider,

$$\frac{V_{out}}{V_{in}} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC}.$$

The magnitude and phase of the frequency response of the low-pass filter are shown in Figure ###. The magnitude response is shown on a log-log scale, whereas the phase response is shown on a linear-log scale. For $\omega >> RC$, the denominator of the transfer function is much greater than one and the input is significantly attenuated. On the other hand, for, $V_{out} \cong V_{in}$. The transition point between the high and low frequency regions is defined when $\omega RC = 1$, where $|V_{out} / V_{in}| = 1/\sqrt{2}$. This is known as the cut-off frequency for the filter,

$$\omega_C = \frac{1}{RC}.$$

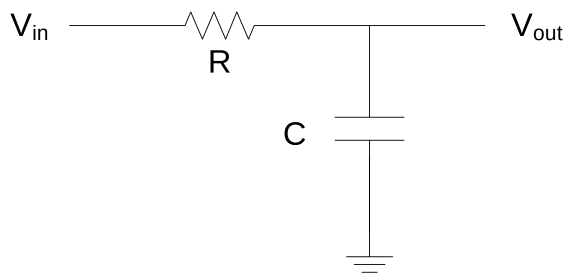

Figure 3.2: Simple RC low-pass filter

###(Insert Figure Here)
Figure ###: The frequency response of a simple RC low-pass filter

A simple high-pass filter can be made by switching the positions of the capacitor and resistor in the low-pass filter. The transfer function is now

$$\frac{V_{out}}{V_{in}} = \frac{R}{R + \frac{1}{j\omega C}} = \frac{j\omega RC}{1 + j\omega RC}.$$

The frequency response of the high-pass filter is shown in *Figure 3.3*. Similar to the low-pass filter, the high-pass filter has a cut-off frequency at $\omega_C = 1/RC$.

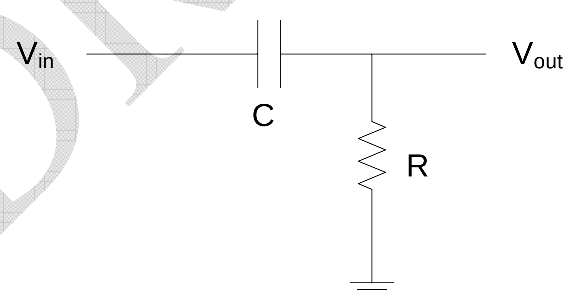

Figure 3.3: Simple RC high-pass filter

###(Insert Figure Here)
Figure ###: The frequency response of a simple RC high-pass filter

3.4 The Impedance of an Inductor

The ratio between voltage and current for an inductor can be found in a similar way as for a capacitor. For a sinusoidal voltage,

$$V_L = Ae^{j\omega t}.$$

Since

$$I_L = L\frac{dV_L}{dt},$$

$$I_L = j\omega LAe^{j\omega t}.$$

Thus, the impedance of an inductor is

$$Z_L = \frac{V_L}{I_L} = j\omega L.$$

Therefore, an inductor is a frequency-dependent resistance that is directly proportional to frequency; Z_L is small at low frequency and large at high frequency. At DC, the impedance of an inductor is zero. Just as for a capacitor, this expression shows that the voltage across an inductor lags the current by a phase of 90^o.

3.5 Simple RL Filters

A low-pass filter can also be made using a resistor and an inductor in series, as shown in *Figure 3.4*. Once again, the transfer function of this filter can be determined like a voltage divider,

$$\frac{V_{out}}{V_{in}} = \frac{R}{R + j\omega L} = \frac{1}{1 + \frac{j\omega L}{R}}.$$

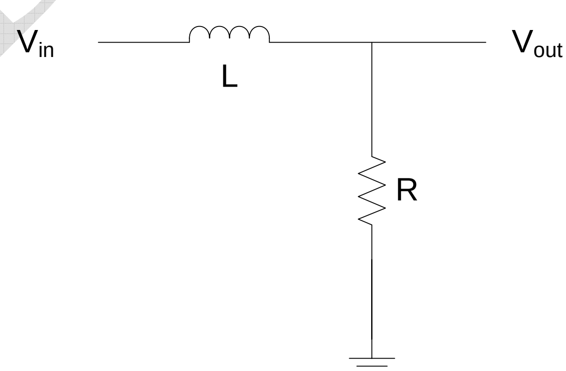

Figure 3.4: Simple RL low-pass filter

###(Insert Figure Here)
Figure ###: The frequency response of a simple RL low-pass filter

The magnitude and phase of the frequency response are shown in Figure ###. For $\omega >> R/L$, V_{out} is attenuated, whereas for $\omega << R/L$, $V_{out} \cong V_{in}$. At the cut-off frequency,

$$f_c = \frac{\omega c}{2\pi} = \frac{R}{2\pi L},$$

$$\frac{\omega L}{R} = 1, \text{ and}$$

$$\left| \frac{V_{out}}{V_{in}} \right| = \frac{1}{\sqrt{2}}.$$

A high-pass RL filter can be made from the low-pass RL filter by switching the position of the inductor and resistor as shown in *Figure 3.5*. The transfer function is

$$\frac{V_{out}}{V_{in}} = \frac{j\omega L}{R + j\omega L} = \frac{\frac{j\omega L}{R}}{1 + \frac{j\omega L}{R}}.$$

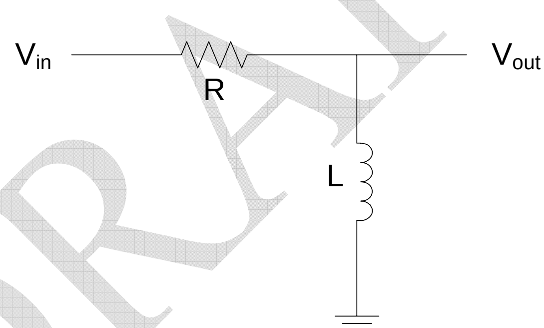

Figure 3.5: Simple RL high-pass filter

###(Insert Figure Here)
Figure ###: The frequency response of a simple RL high-pass filter

The frequency response of the filter is shown in Figure ###. The cut-off frequency of the high-pass filter is the same as the low-pass filter (Eq. ###)

3.6 *s*-Domain Analysis

The concept of complex impedance introduces a unified representation for resistors, capacitors, and inductors, whereby a circuit's frequency response from input to output can be determined using KVL and KCL, where each element is assigned the appropriate impedance. The key assumption to this point is that the input to the circuit must consist solely of DC and/or sinusoidal signals. Now, this analysis is be extended to include arbitrary input signals by using the mathematical techniques of Laplace transforms.

The "brute force" method for determining the response of a circuit to an arbitrary signal is to write a system of linear differential equations using the voltage and current variables in the circuit and then to solve for the output signal, using the input signals as the forcing functions. The Laplace transform simplifies this process by converting linear differential equations in the time domain to algebraic equations in the complex frequency domain. The independent variable in the complex frequency domain is *s*, where

$$s = \sigma + j\omega .$$

The process for solving differential equations using Laplace transforms involves the following general procedure:

1. Write a set of differential equations to describe the circuit;
2. Laplace transform the differential equations to algebraic equations in *s*-domain;
3. Solve for the transfer function in the *s*-domain;
4. Laplace transform the input signal and multiply this by the transfer function to give the output signal in the *s*-domain;
5. Inverse Laplace transform the output signal to give circuit response as a function in the time domain.

The determination of the transfer function in steps 2 and 3 of this procedure can be greatly simplified by Laplace transforming of the impedances of individual circuit elements instead of generalized differential equations that govern circuit behavior. The transfer function can then be found by applying KVL and KCL simplifications to the resulting "Laplace circuit".

For example, the current-voltage relationship of a capacitor is $I_C = C\frac{dV_C}{dt}$ and the Laplace transformed result is $I_C = sCV_C$. Therefore, the impedance of a capacitor in s-domain is $1/sC$. Similarly for an inductor, $V_L = L\frac{dI_L}{dt}$ and the Laplace transform of this is $V_L = sLI_L$. Therefore, the s-domain impedance for an inductor is sL. For a resistor, the s-domain impedance is still *R*. A summary of the s-domain representation of electronic circuits is shown in *Table 3.1*. Interestingly, the s-domain impedance very closely resembles the complex impedances discussed previously. In fact, the s-domain impedance is an extended version of the complex impedance that generalizes to arbitrary signals.

Time Domain Parameter	s-Domain Impedance
R	*R*
C	*1/sC*
L	*sL*

Table 3.1: Summary of s-domain impedances

The impedance representation once again unifies resistors, capacitors, and inductors as equivalent circuit components with specific impedances and the *s*-domain transfer function can be found by using KVL and KCL. The transfer function is generally expressed as a ratio of polynomials such that

$$\frac{V_{out}}{V_{in}} = \frac{Z(s)}{P(s)},$$

where

$$Z(s), P(s) = a_1 s^n + a_2 s^{n-1} ... + a_{n-1} s + a_n .$$

The polynomial can be factored into a number of roots. The roots of the denominator polynomial are known as poles, while the roots of the numerator polynomial are known as zeros.

The frequency response of the circuit is obtained by substituting jω for *s* in the transfer function. The time-domain response is found by implementing steps 4 and 5 of the general procedure: take the Laplace transform of the input signal and then take the inverse Laplace transform of the output signal. In practice, step 4 can be simplified since the time-domain behavior of a circuit is almost always evaluated in response to a step or ramp voltage input, for which the inverse Laplace transforms can be easily computed or obtained from existing tables. Step 5 is also usually simplified since most transfer functions can be approximated by one of a few transfer functions which have known time-domain responses to step and ramp input signals.

3.7 *s*-Domain Analysis Example

3.8 Simplification Techniques for Determining the Transfer Function

3.8.1 Superposition

3.8.2 Dominant Impedance Approximation

3.8.3 Redrawing Circuits in Different Frequency Ranges

4 Source and Load

The ideas of electrical source and load are extremely useful constructs in circuit analysis since all electronic circuits can be modeled as a source circuit, a load circuit, or some combination of the two. Source circuits are circuits that supply energy while load circuits are circuits that dissipate energy. Load circuits can be simply modeled by a single equivalent impedance, while source circuits can be modeled as a voltage or current source plus an equivalent impedance. This section describes the properties of practical voltage and current sources; how to represent the output of arbitrary circuits as source circuits; and how the source and load model of electronic circuits can be used to model circuit behavior.

4.1 Practical Voltage and Current Sources

As discussed in section 1.3, an ideal voltage source will maintain a given voltage across a circuit by providing any amount of current necessary to do so; and an ideal current source will supply a given amount of current to a circuit by providing any amount of voltage output necessary to do so. Of course, ideal voltage sources and ideal current sources are both impossible in practice. When a very small resistive load is connected across an ideal voltage source, a practically infinite amount of current is required. Correspondingly, when a large resistive load is connected across an ideal current source, an exceedingly large voltage is required.

A practical voltage source is modeled as an ideal voltage source in series with a small source resistance, as shown in *Figure 4.1*. The output voltage across the load resistance is attenuated due to the source resistance and the resulting voltage is determined by the resistive divider,

$$V_{out} = V_S \frac{R_{LOAD}}{R_{LOAD} + R_{SOURCE}}.$$

A practical voltage source can approach an ideal voltage source by lowering its source resistance. Therefore, ideal voltage sources are said to have zero source resistance.

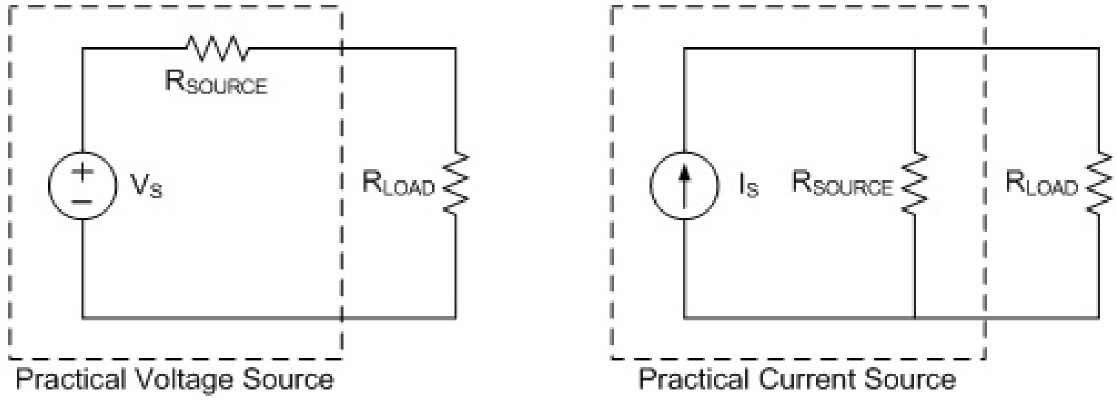

Figure 4.1: Practical voltage and current sources

A practical current source is modeled as an ideal current source in parallel with a large source resistance. The output current is reduced due to the source resistance by the current divider such that,

$$I_{out} = I_S \frac{R_{SOURCE}}{R_{LOAD} + R_{SOURCE}}.$$

A practical current source can approach an ideal current source by increasing its source resistance. Therefore, ideal current sources are said to have infinite source resistance.

4.2 Thevenin and Norton Equivalent Circuits

The practical voltage and current source model can be used to model arbitrary linear source circuits using a technique known as Thevenin and Norton equivalent circuits.

Thevenin's theorem states that the output of any circuit consisting of linear components and linear sources can be equivalently represented as a single voltage source, V_{TH}, and a series source impedance, Z_{TH}, as shown in *Figure 4.2*.

Norton's theorem states that the output of any circuit consisting of linear components and linear sources can be equivalently represented as a single current source, I_N, and a parallel source impedance, Z_N, also shown in *Figure 4.2*.

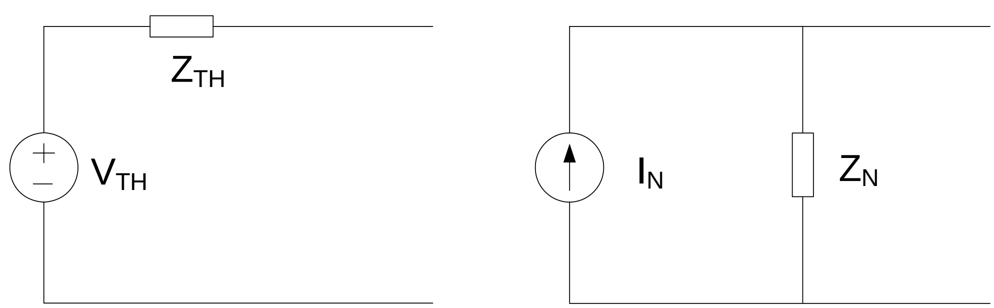

Figure 4.2: Thevenin and Norton Equivalent Circuits

There are simple procedures to determine V_{TH}, I_N, Z_N, and Z_{TH} for a given circuit. To determine V_{TH}, set the load as an open circuit. The voltage across the output is V_{TH}. To determine I_N, set the load as a short circuit, and then the current through the short circuit is I_N. An important link between Thevenin and Norton equivalent circuits is that Z_{TH} and Z_N are exactly the same value. To determine Z_{TH} or Z_N, short-circuit the voltage sources and open-circuit the current sources in the circuit. Z_{TH} or Z_N is then the resulting equivalent impedance of the circuit.

Need Example circuit here with procedure for determining Thevenin and Norton equivalent circuits.###

4.3 Source and Load Model of Electronic Circuits

The simple model of source and load, shown in *Figure 4.3*, is an extremely useful way to predict how two circuits will interact when connected together. The source part can be used to represent the output of a circuit, while the load can be used to represent the input of another circuit. The value of Z_{LOAD} can be determined from the equivalent circuit model of the load circuit using KVL and KCL. The value of Z_{SOURCE} can be determined by Thevenin and Norton equivalent circuits. The voltage across the load can simply be calculated from,

$$V_{LOAD} = V_{SOURCE} \frac{Z_{LOAD}}{Z_{LOAD} + Z_{SOURCE}}$$

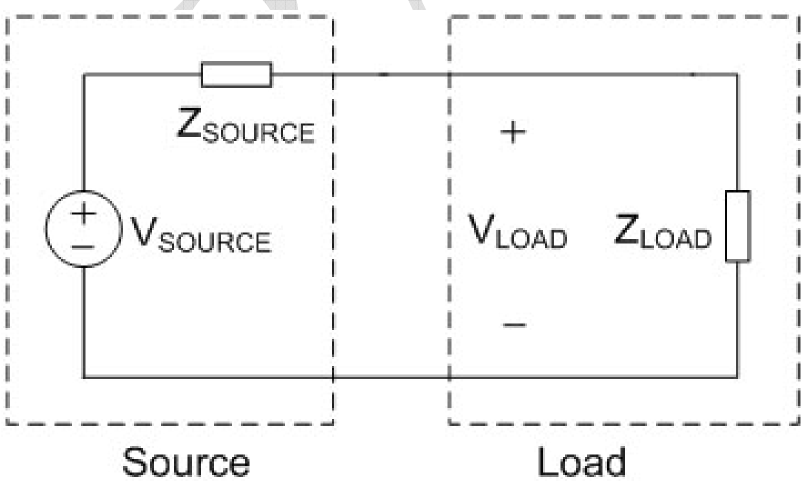

Figure 4.3: Source and load model

The source and load model is particularly useful when the design of either the source or load circuit is fixed and a circuit is required to be the corresponding source or load circuit. For example, if the output of a voltage amplifier behaves as a voltage source with a source resistance of 500Ω, the input resistance of the next circuit, being the load impedance, should be significantly greater than 500Ω in order to prevent the source voltage from being attenuated.

5 Critical Terminology

As with any engineering discipline, electrical engineering is full of its own special words and lingo that can make electrical engineering speak sound like a foreign tongue. Buffer, bias, and couple are three such words that can often trip-up new comers.

5.1 Buffer

Buffer is one of those words that seem to have a different meaning in every discipline of science and engineering. Buffer has two meanings in electrical engineering depending if the context is analog or digital electronics.

In analog electronics, to buffer means to preserve the content of a low power signal and convert it to a higher power signal via a buffer amplifier. This is a frequent operation in analog electronics since low power signals can be more easily interfered with than high power signals, but often only low power signals are available from electronic sensors.

If signals are represented by voltage in a circuit, the power of the signal is proportional to the amount of current drawn by the circuit. Since current draw is dependent on the impedance of the circuit, a high impedance circuit has less power, while a low impedance circuit has more power. The function of a buffer amplifier in this case is to convert a high impedance circuit to a low impedance circuit. This buffering scenario is represented by an equivalent circuit shown in *Figure 5.1* where a voltage output electronic sensor has relatively high output impedance Z_{OUT1}. If the sensor output is used to drive a load impedance, Z_{LOAD}, directly, much of the voltage signal may be lost to attenuation. In order to remedy this problem, a buffer amplifier is inserted between the sensor output and Z_{LOAD}. The input of the buffer amplifier measures this voltage signal with a high input impedance Z_{IN}, and replicates the signal V_{IN} with an output impedance Z_{OUT2}. Since Z_{OUT2} is smaller than Z_{OUT1}, the sensor signal can be used to drive Z_{LOAD} without suffering significant attenuation.

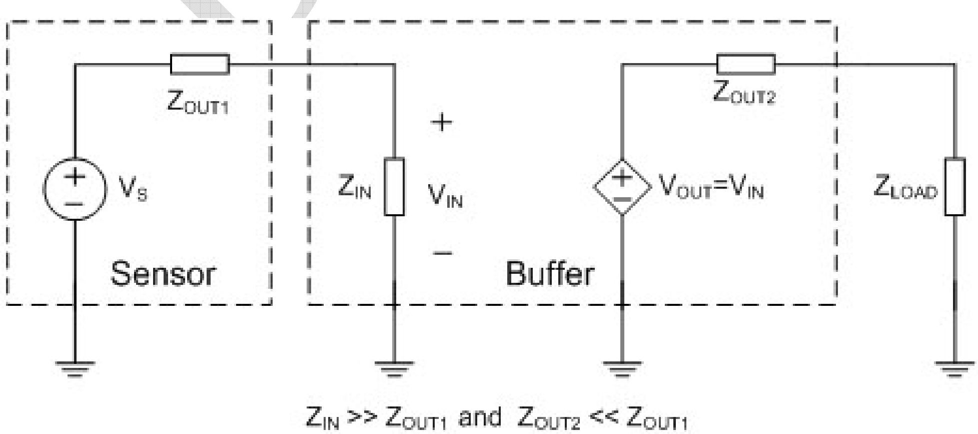

Figure 5.1: A Functional Equivalent of a Buffer in Analog Electronics

In digital electronics, buffer refers to a mechanism in the communications link between two devices. When there are time-lags between the transmitting device and the receiving device, some temporary storage is necessary to store the extra data that can accumulate. This temporary storage mechanism is known as a buffer. For example, there is a buffer between the keyboard and the computer so that the CPU can finish one task before accepting more input to initiate another task. Digital video cameras have a much larger buffer to accumulate raw data from the camera before the data can be compressed and stored in a permanent location.

5.2 Bias

Bias refers to the DC voltage and current values in a circuit in absence of any time-varying signals. In circuits that contain nonlinear components such as transistors and diodes, it is usually necessary to provide a power supply with static values of voltage and current. These static operating parameters are known as the bias voltage and bias current of each device. When analyzing circuit response to signals, the circuit components are typically linearized about their DC bias voltage and current and the input signals are considered as linear perturbations. Frequently, the AC behavior of a circuit component is dependent on its bias voltage and bias current.

5.3 Couple

The word *couple* means to connect or to link a signal between two circuits. There are generally two types of coupling: DC and AC. As shown in *Figure 5.2*, DC coupling refers a direct wire connection between two circuits; while AC coupling refers to two circuits connected via a capacitor. Between AC coupled circuits, signals at frequencies below some cut-off frequency will be progressively attenuated at lower frequencies. The cut-off frequency is determined by the coupling capacitance along with the output impedance of the transmitting circuit and the input impedance of the receiving circuit. Oscilloscope inputs also have both AC and DC coupling options, which allow the user the select between viewing the total signal or just the time-varying component. In addition to describing intentional circuit connections, AC coupling also refers to the path of interference signals through stray capacitances in the physical circuit. There are also a number of other coupling mechanisms not included in this discussion, such as magnetic, optical, and radio-frequency coupling.

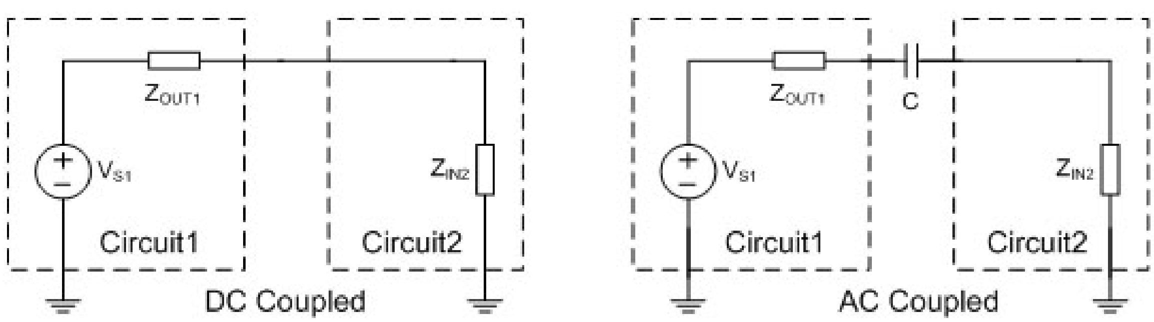

Figure 5.2: DC and AC coupled circuits

6 Diodes

6.1 Diode Basics

A Diode is an electronic equivalent of a one-way valve; it allows current to flow in only one direction. There are two terminals on a diode, which are known as the anode and cathode. Current is only allowed to flow from anode to cathode. The symbol and drawing for the diode are shown in *Figure 6.1* and *Figure 6.2*. The dark band of the diode drawing indicates the cathode mark on the diode symbol. The direction of current flow is indicated by the direction of the triangle. An easy trick for remembering the direction of current flow is to remember that of the current flowing *alphabetically*, from the *anode* to the *cathode*.

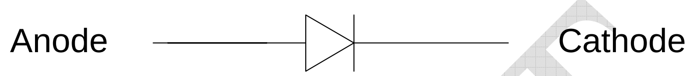

Figure 6.1: Circuit symbol for a diode

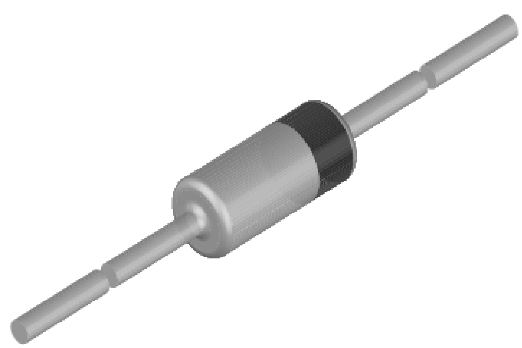

Figure 6.2: 3D model of a diode, the dark band indicate cathode (Courtesy of Vishay Semiconductors)

The unidirectional conduction through a diode is explained by semiconductor physics. A diode is a junction between N-type and P-type semiconductors, typically fabricated in thin layers as shown in *Figure 6.3*. Both N-type and P-type materials are electrically neutral, but have different mechanisms of conduction. In N-type material, negatively charged electrons are mobile and are the majority current carriers. In P-type material, positively charged holes are mobile are the majority current carriers. Holes are actually temporary positive charges created by the lack of an electron; the details of this interpretation can be found in textbooks on semiconductor physics [Ref: Sedra and Smith].

Near the junction interface, electrons from the N-type region diffuse into the P-type region while holes from the P-type region diffuse into the N-type region. The diffused electrons and holes remain in a thin boundary layer around the junction known as the depletion layer shown in *Figure 6.3*. The excess of positive and negative charges create a strong electric field at the junction which acts as a potential barrier that prevent electrons from entering the P-type region and holes from entering the N-type region.

When a negative electric field is applied from anode to cathode, the depletion region enlarges and it becomes even more difficult to conduct current across the junction. This is the reverse-conducting state. When a positive electric field is applied from anode to cathode, the depletion region narrows and allows current to conduct from the anode to the cathode. This is the forward-conducting state.

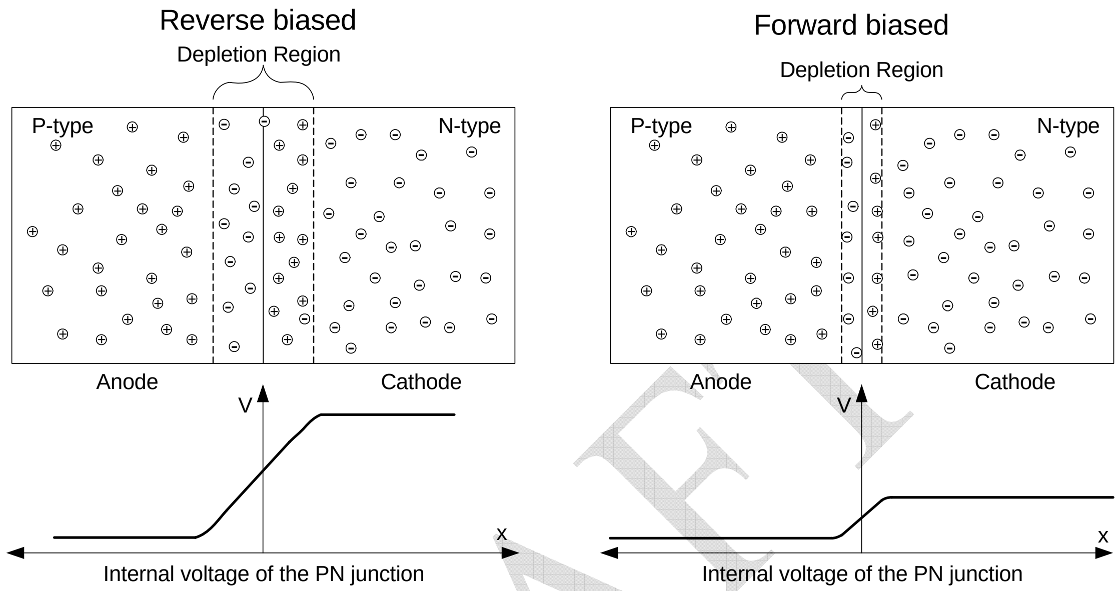

Figure 6.3: PN Junction in a diode showing the depletion region

The current-voltage relationship for a diode is shown in *Figure 6.4*. The current is an exponential function of voltage such that

$$I(V) = I_o\left(e^{\frac{V}{V_t}} - 1\right), \text{ and}$$

$$V_t = \frac{kT}{Q}.$$

At 300 K room temperature, V_t = 26 mV. This means that for every increase of V_t in voltage, the current drawn by the diode scales by a factor of e. For most design purposes, the detailed exponential behavior of a diode can be approximated as a perfect conductor above a certain voltage and a perfect insulator below this voltage (*Figure 6.4*). This transition voltage is known as the "knee" or "turn-on" voltage. For silicon (Si) diodes, the knee voltage is 0.7 V; for Schottky barrier and germanium (Ge) diodes, the knee voltage is 0.3 V; and for gallium-arsenide (GaAs), gallium-nitride (GaN), and other hetero-junction light-emitting diodes, the knee voltage can range from 2 to 4 V.

When a reverse voltage is applied to a diode, the resistance of the diode is very large. The exponential current-voltage behavior predicts a constant reverse current that is approximately equal to I_o. In reality, the mechanisms responsible for reverse conduction are leakage effects which are proportional to the area of the PN-junction. Silicon diodes typically have maximum reverse leakage currents on the order of 100 nA at a reverse voltage of 20 V, while the leakage currents for germanium and Schottky barrier diodes can be much higher.

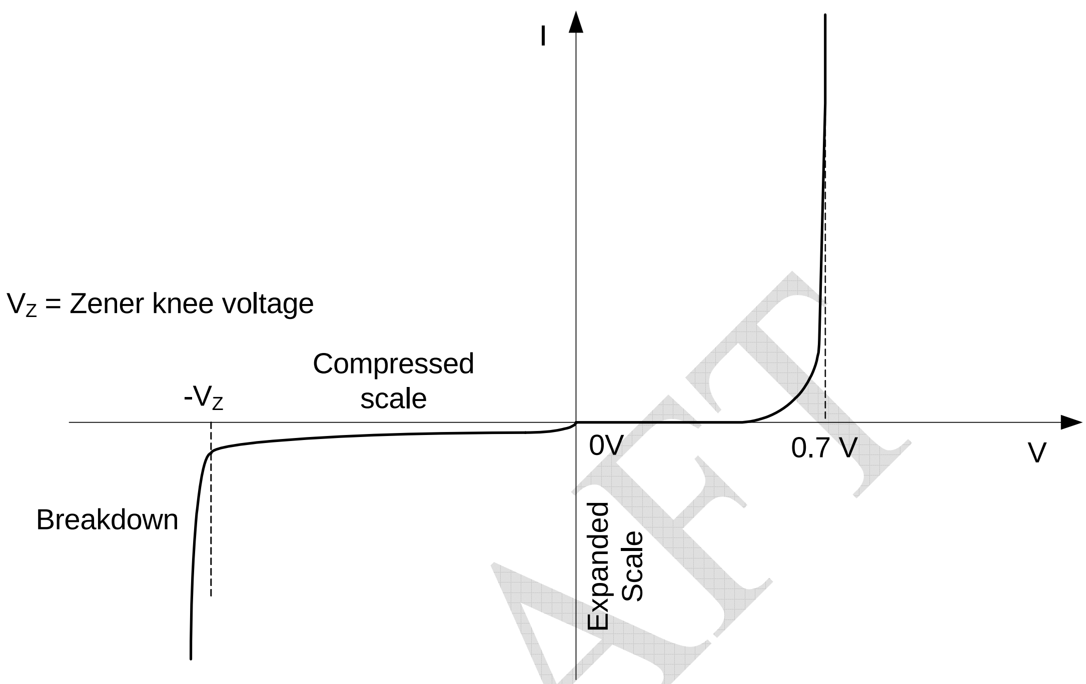

Figure 6.4: Diode current-voltage relationship (not to scale)

When a sufficiently high reverse voltage is applied across a diode, electrical breakdown can occur across the PN-junction, resulting in massive conduction. In some cases, this effect is reversible and if used properly, will not damage the diode. This is known as the Zener effect and such diodes can be specifically engineered to create a similar current-voltage non-linearity in the reverse direction as in the forward direction. In fact, the knee in the current-voltage relationship for Zener diode can be significantly sharper and can range from 1.8V to greater than 100V. The symbol for a Zener diode is shown in *Figure 6.5*. In practical electronic circuits, Zener diodes are often used to make voltage references as well transient voltage suppressors.

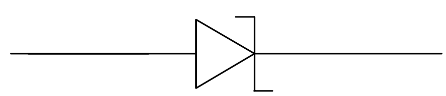

Figure 6.5: Symbol for a Zener diode

6.2 Diode Circuits

6.2.1 Peak detector

A classic diode circuit is a peak detector shown in circuit a, *Figure 6.6*, having a diode and a capacitor in series. On the upswing of the signal, when the source voltage (V_S) is 0.7 V greater than the capacitor voltage (V_C), the diode has a small resistance and $V_C = V_S - 0.7$. On the downswing of the signal, the diode has a large resistance and the previous peak voltage is held on by the capacitor. If the signal is kept consistently lower than the capacitor voltage, then the capacitor voltage decays with a time constant that is equal to the reverse diode resistance multiplied by the capacitance. Since the reverse diode resistance can be as large as 10^9 ohms, the decay time constant may be undesirably long. If this is the case, a large resistor can be added in parallel with the capacitor, as shown in circuit b, *Figure 6.6*, to set the decay time constant as $\tau = RC$.

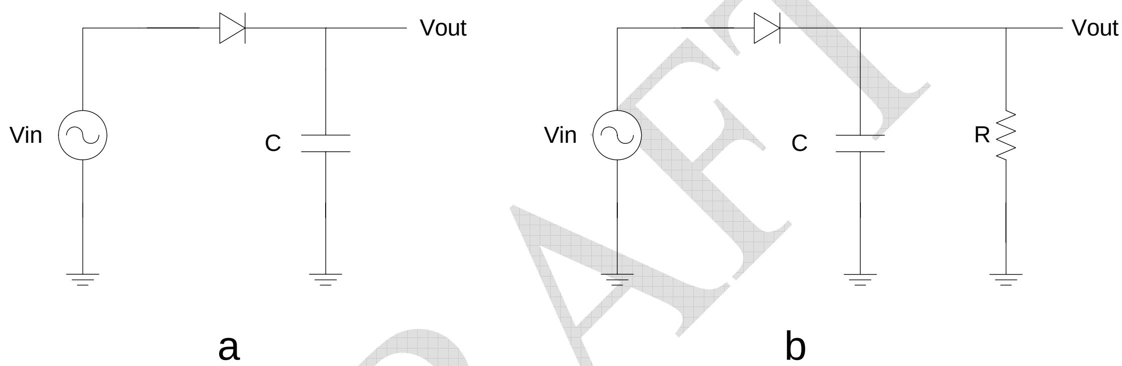

Figure 6.6: Diode peak detectors

6.2.2 LED Circuit

One of the most useful types of diodes is the light-emitting diode (LED). When current flows in the forward direction, an LED emits light proportional to the amount of forward current. Recent advances in semiconductor materials have drastically increased the power, efficiency, and range of colors of LED's. It will not be long before many traditional lighting devices, such as fluorescent lights, are replaced by bright LED's.

LED's cannot be powered directly from a voltage source as shown in *Figure 6.7*, circuit a, due of the sensitivity of their current-voltage relationship. Since mV differences in the applied voltage can drastically alter their operating current, manufacturing variations would make it impossible to control their current flow this way. When an LED is powered using a voltage source, a current-limiting resistor should be used, as shown *Figure 6.7*, circuit b. For example, suppose that the turn-on voltage of the LED is 2.1V and the voltage source is the output of a 3.3V microprocessor. The resistor is chosen to operate the LED at the specified maximum operating current of 10mA, according to the following simple analysis. Given the 2.1V turn-on voltage drop, 3.3 - 2.1 = 1.2 V will be dropped across the resistor. The desired operating current is 10mA, and therefore the necessary resistance is 1.2 V / 10 mA = 120 Ω.

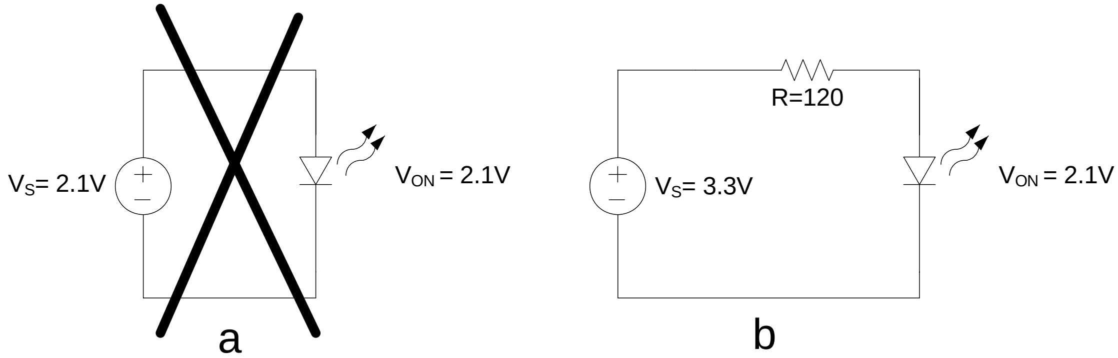

Figure 6.7: Circuit for powering LED's

While this method for powering a LED is simple and ubiquitous, it is inefficient since the power applied to the resistor is dissipated as heat. In the above example, more than one-third of the applied power is lost. Efficiency can be increased by using specialized integrated circuits that drive LED's using a constant current source.

6.2.3 Voltage Reference

The highly non-linear current-voltage relationship of diodes also make them ideal for making voltage references. As shown in *Figure 6.8*, silicon diodes biased in the on-state can be used to generate a 0.7V reference. For references from 1.8V and above, Zener diodes biased in the reverse direction can be used in a similar circuit. The output current sourcing capacity is limited by the bias current through the diode. In order to maintain the correct bias in the diode, the bias current through diode should be at least a few mA greater than the maximum current that the reference will be required to source. The resistor value can be selected using the same procedure for the LED circuit.

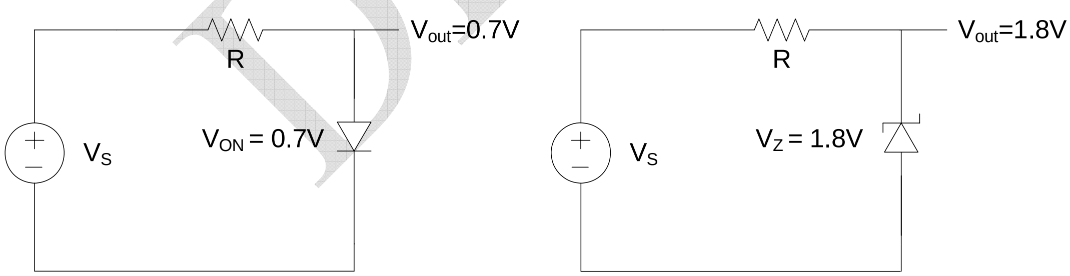

Figure 6.8: Voltage reference circuits using a diode or Zener diode

7 Transistors

Transistors are active non-linear devices that facilitate signal amplification. In the hydrodynamic model of electrical current, transistors are equivalent to a dam with a variable gate that controls the amount of water flow shown in *Figure 7.1*. Just as in a real dam, a small amount of energy is required to operate the gate. Amplification is achieved in the sense that a small amount of energy can be used to control the flow of a large amount of current.

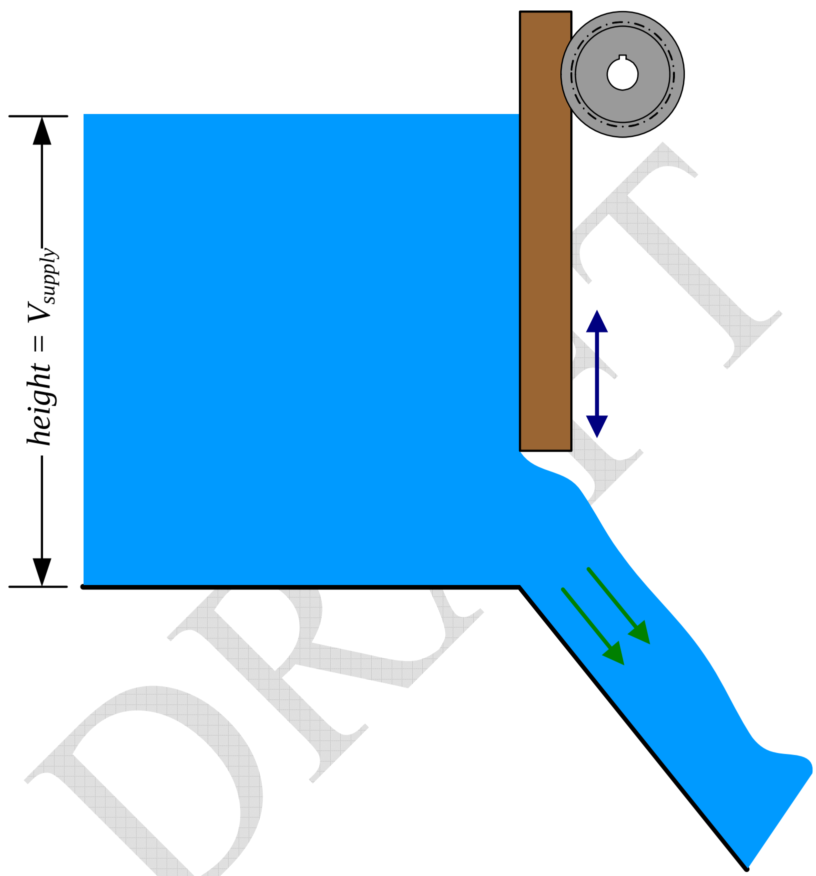

Figure 7.1: The hydrodynamic analogy of a transistor

There are two main classes of transistors: bipolar-junction transistors and field-effect transistors.

7.1 Bipolar-Junction Transistors (BJT)

A BJT has three terminals: emitter, base, and collector. In the hydrodynamic analogy, the emitter and collector correspond to the river above and below the dam. The base terminal corresponds to the control input that varies the flow through the dam.

There are two varieties of BJT's: NPN devices that use electrons as the primary charge carrier and PNP devices that use holes as the primary charge carrier. The circuit symbols for NPN and PNP BJT's are shown in *Figure 7.2*. From this point on, the discussions of BJT behavior will use NPN devices as examples. The discussion for PNP devices is exactly complementary to that of NPN devices except electrons and holes are interchanged and as a result, all the characteristic device voltages are reversed.

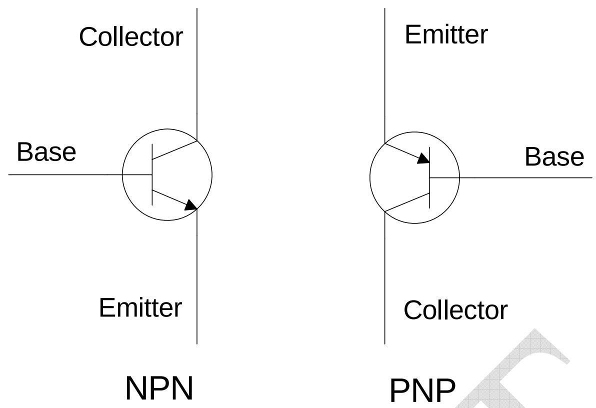

Figure 7.2: Circuit symbols for the NPN and PNP transistor

The structure of an NPN BJT, shown in Figure ###, consists of three layers of materials: an N-type layer, a thin P-type layer, and another N-type layer, which corresponds to the emitter, base, and collector. The emitter-base and collector-base PN-junction form diodes with opposing directions of conduction as shown in *Figure 7.3*. Typically, the collector is connected to a higher voltage than the emitter, while the base is connected to a voltage between the two. The collector-to-emitter voltage is equivalent to the height of the water in the dam model. The base voltage is equivalent to the position of the control gate. When the voltage at the base is not enough to turn-on the base-emitter diode, there is no conduction from collector to emitter. When the voltage at the base is high enough to turn-on the base-emitter diode, a conduction path from collector to emitter is opened.

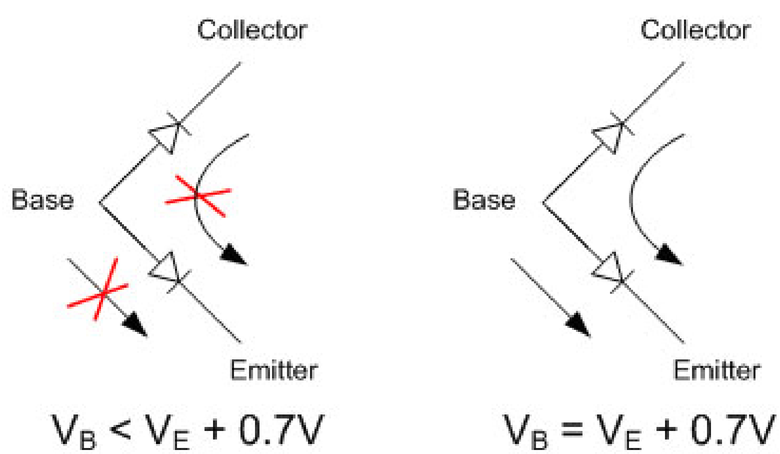

Figure 7.3: A crude model of a NPN BJT

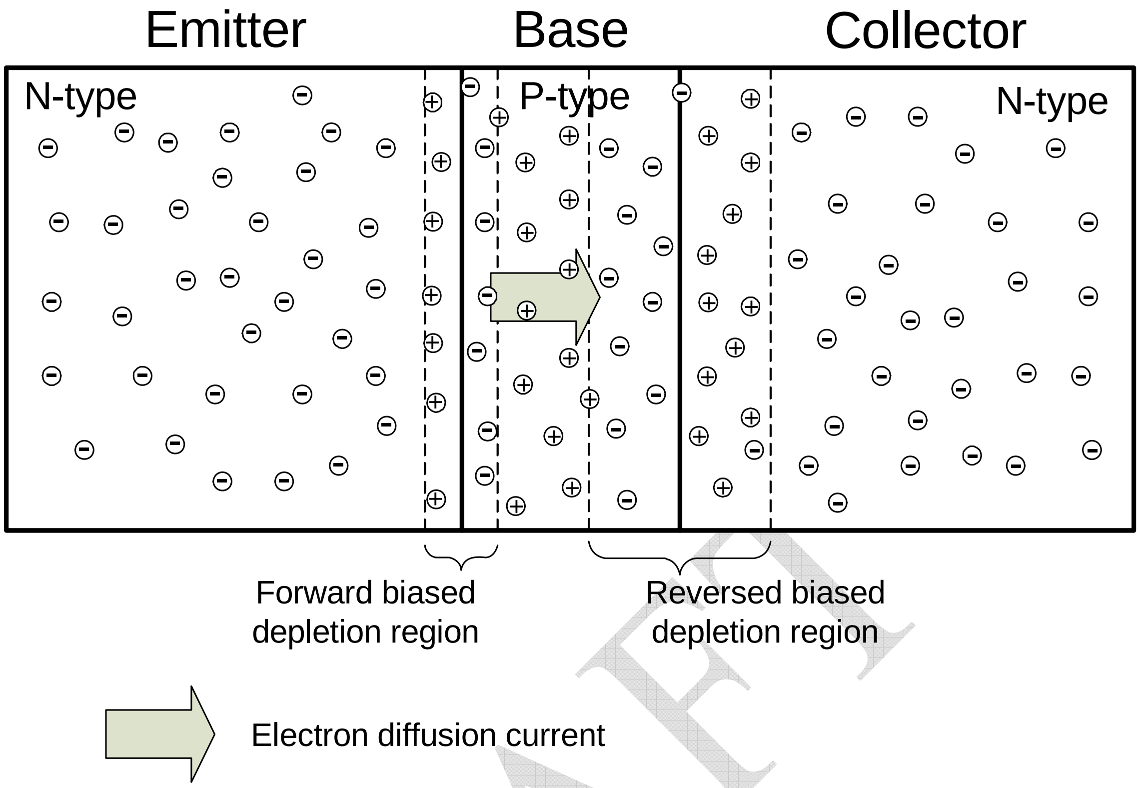

Figure 7.4: Conceptual structure of an NPN transistor

The mechanism responsible for controlling the conductivity from collector to emitter lies in the depletion layers formed at base-emitter and base-collector PN-junctions shown in *Figure 7.4*. When the base-emitter PN-junctions is reverse biased, electrons from the emitter are unable to get across the potential barrier of the PN-junction. When the base-emitter diode is forward biased, electrons from the emitter are injected into the base. Since the base is very thin, the electrons can easily diffuse across and be "swept up" by the electric field in the reverse-biased depletion layer of the base-collector junction. This electron diffusion current from the emitter to the collector is the large output current of a BJT.

A simple way to visualize the conduction mechanism of BJT is to consider the electrons introduced into the thin base region as effectively changing base material from P-type to N-type. Therefore, the transistor structure changes from the non-conductive N-P-N layers to the conductive N-N-N layers.

The amount of diffusion current is approximately proportional to the amount of base-emitter current. The proportionality constant, typically denoted as β, is the current gain of the transistor. The value of β depends on geometry of the device and the doping concentrations of the collector, base, and emitter. For most commercial transistors, β is between 100 and 1000. A much more detailed explanation of the physics and design of transistors can be found in texts such as [Ref].

As a circuit element, a BJT can be modeled as a combination of a diode (from base to emitter) and a voltage-dependent current source (from collector to emitter) as shown in *Figure 7.5*. The diode has an exponential current-voltage relationship and draws current equal to the base current,

$$I_b = I_{b_o}\left(e^{\frac{V_b}{V_t}} - 1\right).$$

The current source represents the collector to emitter current, $I_c = \beta I_{b_o}$. When the transistor is turned on, the base-emitter voltage is constrained by the diode, while the collector-emitter voltage can vary because the current source will produce the necessary voltage to draw the desired amount of current. In practice, however, the collector-emitter voltage must be greater than a few hundred milli-volts to avoid forward-biasing of the base-collector diode.

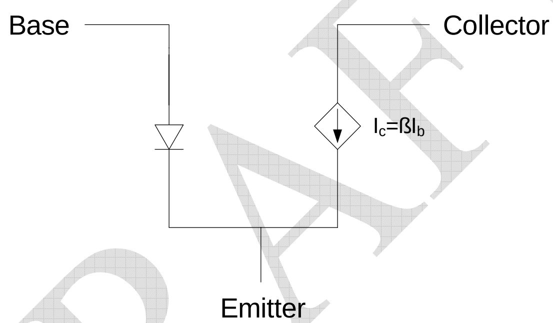

Figure 7.5: Circuit representation of a BJT

In circuits involving discrete transistors, BJTs are often used as a switch to turn on and off currents to components such as LED's. Figure ### shows four such configurations where NPN and PNP transistors are used to switch on and off loads connected to a 5V power supply. In each configuration, the input signal is connected to the base via the current-limiting resistor (R_b). Since the base-emitter junction behaves as a diode, the voltage across the base-emitter junction is effectively held at 0.7V. The function of R_b is to limit the amount of current between base and emitter to prevent burning out the transistor. Typically, values ranging from 500 Ω to 50kΩ are acceptable.

7.2 Field-Effect Transistors (FET)

Field-effect transistors are far more ubiquitous than BJTs in today's integrated electronic circuits. The most common FET is the metal-oxide semiconductor field-effect transistor (MOSFET), millions of which are found in each computer CPU. Similar to NPN and PNP type BJTs, there are also N-channel and P-channel MOSFETs. A circuit containing both N- and P-type MOSFETs is called a complementary-MOS circuit, giving the widely-used acronym CMOS.

The MOSFET device has four-terminals including gate, source, drain, and body as shown in *Figure 7.6*. The gate is a metal electrode that is insulated from the other three terminals via a thin oxide layer. In an N-channel MOSFET, the source and drain are N-type and the body is P-type. Conversely, for a P-channel MOSFET, the source and drain are P-type and the body is N-type. The N-channel device will be used as an example to explain the basic principles of the MOSFET.

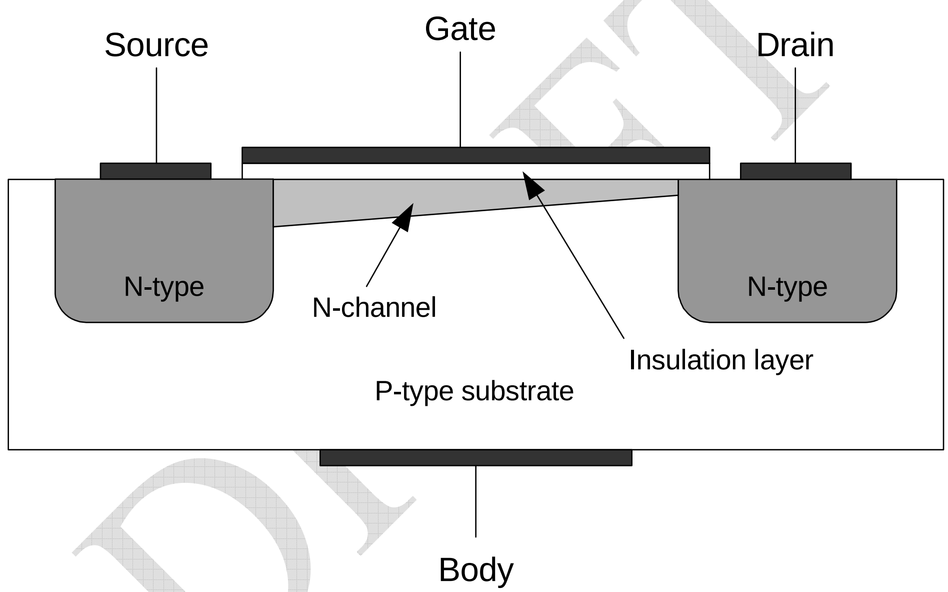

Figure 7.6: Structure of an N-channel MOSFET

The conduction path in a MOSFET runs from drain to source. Similar to a BJT, when a MOSFET is in the off-state, the body-drain and body-source junctions form opposing diodes and the conduction path is blocked. In a BJT, the conduction path is created by introducing electrons to the middle P-type region to temporarily turn it into an N-type region. In a MOSFET, charges which are deposited at the gate electrode create an electric field in the body. This field draws electrons into a thin layer in the body adjacent to the oxide layer. The excess of electrons effectively creates an N-type channel in the P-type body, which gives a high drain-to-source conductivity.

The gate-body voltage required to create a conductive channel, known as the threshold voltage, V_{th}, depends on the thickness of the oxide layer and on the doping level in the body. For N-channel MOSFET's, V_{th} typically ranges from 2 V to 10 V and for P-channel MOSFET's, V_{th} ranges from -2 V to -10 V. During operation, once a channel is created, the drain-source current varies proportionally with the square of the difference between the gate voltage and V_{th},

$$I_d = K\left(V_{gs} - V_{th}\right)^2,$$

where K is a device-dependent conductivity parameter.

While a MOSFET is essentially a symmetric device between source and drain, the body must be connected to the lowest voltage in order to prevent forward-biasing the PN junction between the body to either drain or source. By convention, the body is connected to the source.

The circuit symbols for the N-channel and P-channel MOSFET's are shown in *Figure 7.7*. Since the body is connected to the source in most instances, the body contact is sometimes omitted as shown.

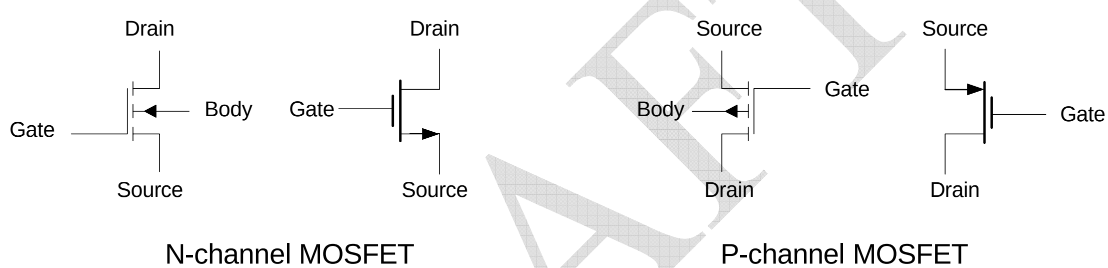

Figure 7.7: N-channel and P-channel MOSFET circuit symbols with and without body contact

The equivalent circuit model for a MOSFET is shown in Figure ###. In DC operation, the gate is an infinite impedance sense while the drain-source current is determined by a voltage-dependent current source. This model reveals an important difference between MOSFET's and BJT's. Unlike BJT's, a MOSFET does not require a DC current to be turned on. However, a MOSFET does have a significant gate to body capacitance and requires a transient current to deposit enough charges at the gate to create a conductive channel from drain to source.

Figure ###: Equivalent circuit model for a MOSFET

Need examples of MOSFET circuits

8 Operational Amplifiers

8.1 Op amp Basics

Operational amplifiers (op amps) are pre-packaged transistor amplifier building blocks designed for analog signal processing. Its name is a legacy of its original purpose to perform arithmetic operations in analog computing. First sold as a monolithic component in the 1960's, op amps have proven to be the most versatile building block in analog circuit design. Today, the availability of low-cost and high performance of op amps makes them ubiquitous in almost all analog circuits.

Op amp circuits are used to amplify, offset, filter, sum, and buffer analog signals, among many other functions. The non-linear nature of transistors makes these operations difficult to perform without distortion. Op amps are able to avoid this problem by using the mechanisms of negative feedback.

An op amp has five terminals, shown in *Figure 8.1*: the non-inverting input, inverting input, output, positive supply, and negative supply. In this text, the non-inverting input is referred to as the plus terminal, and the inverting input as the minus terminal.

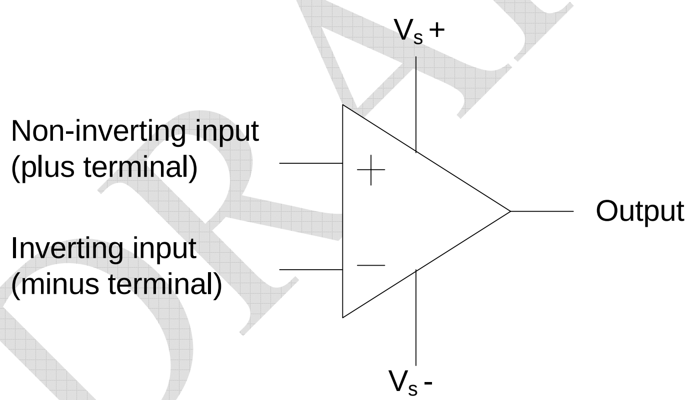

Figure 8.1: Circuit symbol for an op amp

The op amp's power supply terminals are typically omitted from conceptual circuit sketches; although, the power supply is often a source of circuit problems and requires special attention from the designer. Typically, op amps are powered by bipolar supplies such as ±12 V or ±5 V. The op amp's maximum output cannot exceed the supply voltage range, and is usually at least 1V less than either supply limit. When the op amp's output has reached its positive or negative maximum, the output has "railed". This condition should be avoided in most instances.

The internal circuitry in an op amp is designed to give two basic characteristics: (1) very high impedance at the input terminals, and (2) very high differential gain from the input to the output. In fact, an ideal op amp has infinite input impedance and infinite differential gain. For practical op amps, the differential gain exceeds 10^5 and input impedance is a few GΩ. At first, it may be difficult to understand how a very high gain amplifier can be useful since a very small differential voltage will send the output to rail to its maximum or minimum value. However, op amps are designed to operate in feedback, which regulates the output voltage according to the input as a result of amplifier's configuration.

An intuitive grasp of op amps in feedback can be found by examining the voltage follower circuit shown in *Figure 8.2*. The input voltage is applied at the plus terminal while the output voltage is fed back into the minus terminal. The voltage follower will replicate the input voltage at its output while isolating disturbances at the output from affecting the input.

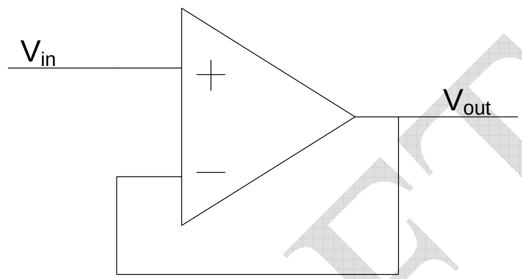

Figure 8.2: Op amp voltage follower circuit

The voltage follower circuit functions in the follow way: if the negative terminal (also the output) is lower than the positive terminal, the gain of the amplifier will make the output more positive and thereby bringing the negative terminal closer to the positive terminal. If the negative terminal is above the positive terminal, the gain of the amplifier will make the output more negative and thereby bringing the negative terminal closer to the positive terminal. From this crude analysis, it is possible to see that regardless of the output starts higher or lower than the input, the feedback mechanism will make the output approach the input voltage. The exact nature of the errors between output and input and the dynamic properties of the voltage signals will be discussed in detail in Section 10 - Feedback.

8.2 Op amp Circuits

The first order analysis of op amp circuits can be made following two simple rules: (1) the plus and minus terminals draw no current; and (2) the output will produce whatever voltage is necessary to equalize the voltages at the plus and minus terminals. The prerequisites for using these rules are that the op amp must be in negative feedback and the required output must be within the valid output range of the op amp.

8.2.1 Non-inverting Amplifier

The non-inverting amplifier circuit is shown in *Figure 8.3*. The input is applied to the op amp's positive input. The output is fed back to the op amp's negative input via the resistive divider formed by R_1 and R_2. Note that for all practical purposes, the amplifier's input impedance is infinite. Since the feedback mechanism ensures that the positive and negative terminals of the op amp are equivalent, the voltages at the inputs are

$$V^{+} = V^{-} = V_{out}\left(\frac{R_1}{R_1 + R_2}\right)$$

Therefore, the transfer function from input to output is

$$\frac{V_{out}}{V_{in}} = \left(\frac{R_1 + R_2}{R_1}\right)$$

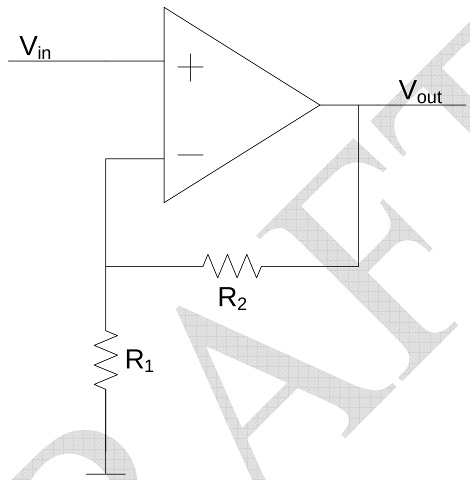

Figure 8.3: Non-inverting amplifier circuit

A special case of the non-inverting amplifier is when R1 is infinite and R2 is 0. This forms the op amp voltage follower as shown previously in *Figure 8.2* which has a net gain of 1.

Non-inverting amplifiers have extremely high input impedance which makes them an excellent buffer for high impedance signals. This is because the input is fed directly to the plus-terminal of the opamp which typically has an input impedance on the order of $10^9\Omega$.

8.2.2 Inverting Amplifier

The inverting amplifier circuit is shown in *Figure 8.4*. The input is applied to the resistor R1 which is connected to the op amp's minus terminal. The plus terminal is tied to ground while the output is fed back to the negative terminal through resistor R_F. Since no current flows into the minus terminal, all current sourced from the source through R1 must be drained through R_F. The feedback mechanism ensures that voltages at the plus and minus terminals are equal, which means that

$$\frac{V_{out}}{R_F} = -\frac{V_{in}}{R_1},$$

and the transfer function is

$$\frac{V_{out}}{V_{in}} = -\frac{R_1}{R_F}.$$

The input impedance of the inverting amplifier is set by resistor R3 and is significantly lower than the input impedance of the non-inverting amplifier.

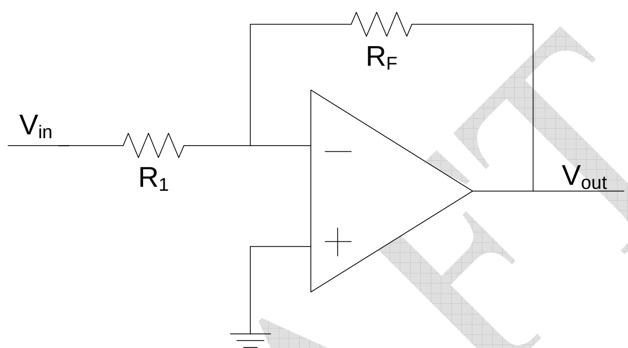

Figure 8.4: Inverting amplifier configuration

This amplifier is sometimes called current-summing amplifier because all current sourced into the negative input will be drain via the feedback resistor. This property can be used to amplify current based signal sources such as photodiodes as shown in Figure ###. In this case the output voltage is simply $V_{out} = R_F I_{in}$. A much more extensive discussion on photodiode amplifiers can be found in section 17.3.2

###Insert simple photodiode amplifier circuit here###
Figure ###: Simple photodiode amplifier

It is also possible to sum signals from multiple sources as shown in *Figure 8.5*, where the output voltage is

$$V_{out} = R_F\left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}\right).$$

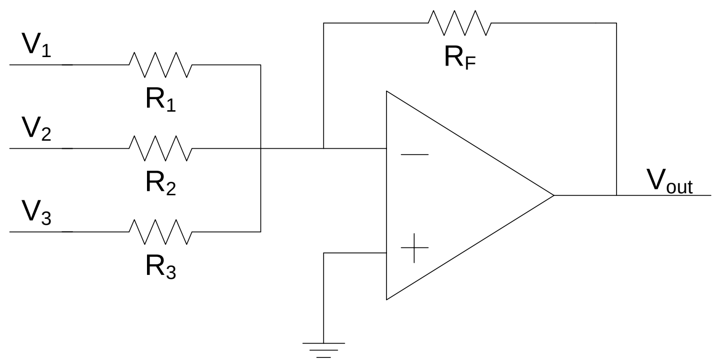

Figure 8.5: Inverting op amp summer

8.2.3 Signal Offset

In analog signal processing, it is often desirable to offset the signal by a known amount. For example, if the signal ranges from -5 V to +5 V and the analog-to-digital converter is designed to accept signals ranging from 0 V to +10 V, a +5 V shift is required. Such signal offsets can be accomplished using an inverting or non-inverting amplifier gain stage and be integrated with modest gain functions.

Figure 8.6 shows how the voltage offset can be accomplished using an inverting amplifier. The positive input is now connected to a reference voltage instead of ground. The transfer function will now need to be re-evaluated with the non-zero reference voltage factored in. The output is

$$V_{out} = V_{REF} - (V_{in} - V_{REF})\left(\frac{R_F}{R_1}\right).$$

The incremental gain of AC signals between V_{out} and V_{in} is still

$$\frac{V_{out}}{V_{in}} = -V_{in}\left(\frac{R_F}{R_1}\right).$$

Since the input of the amplifier draws nearly zero current, the reference voltage can be generated using a simple voltage divider where

$$V_{REF} = V_S \frac{R_3}{R_2 + R_3}.$$

A capacitor is necessary to reduce noise at the reference voltage since any variation of the reference voltage will directly couple onto the output.

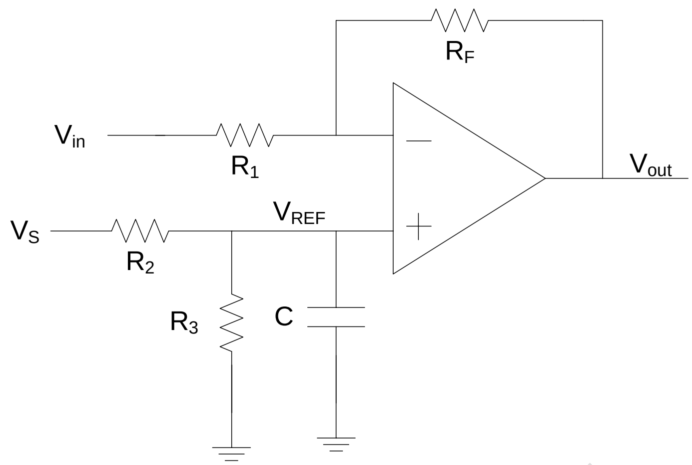

Figure 8.6: Inverting amplifier with voltage offset

Figure 8.7 shows how the voltage offset can be accomplished using a non-inverting amplifier. Resistor R_1 is connected to the reference voltage instead of ground and therefore the output is

$$V_{out} = \left(V_{in} - V_{REF}\right)\frac{R_2 + R_1}{R_1}.$$

A more complicated reference voltage is required since the reference is now required to sink or source current through R_1 while maintaining a constant voltage. In other words, the reference must act like a voltage source with a small output resistance. This voltage reference can be generated using a voltage divider circuit buffered by an op amp voltage follower.

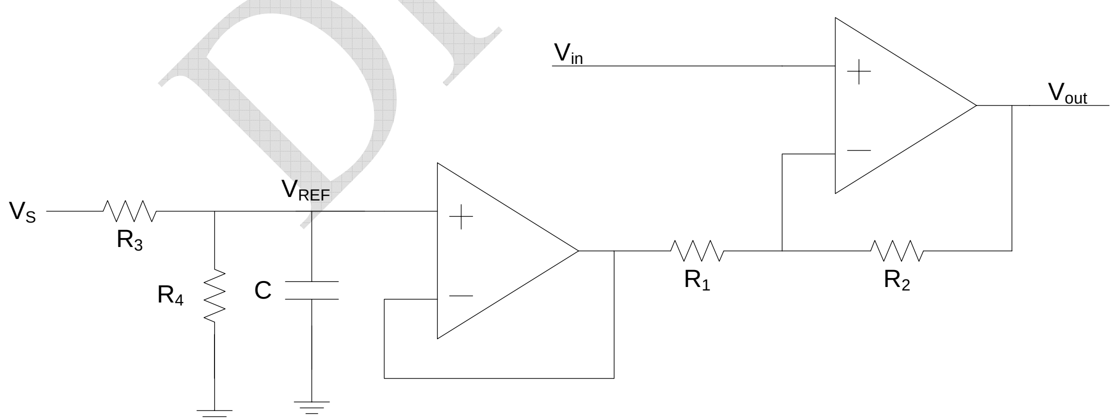

Figure 8.7: Non-inverting amplifier with voltage offset

9 Filters (Very incomplete)

Electronic filters are designed to attenuate signals in one part of the frequency spectrum while emphasizing signals in another part of the spectrum. There are four basic types of filter responses as shown in Figure ###: low-pass, high-pass, bandpass, and band-reject. Low-pass filters are designed to pass signals from DC ($\omega = 0$) to $\omega = \omega_c$, where ω_c is known as the cutoff frequency. High-pass filters are designed to pass signals from ω_c to infinite frequency. Bandpass filters are a combination of low-pass and high-pass filters where the width of the pass band is known as the bandwidth of the filter. Band-reject or notch filters are the inverse of bandpass filters, and are designed to remove interference from a certain frequency range.

Figure ###: Basic filter responses

Electronic filters are created by building circuits that has transfer functions that attenuate signals at certain frequencies. These filter circuits can be either passive or active. Passive filters generally involve resistors, capacitors, and inductors while active filters use resistors, capacitors, and op amps to eliminate the need for inductors. Passive filters are simpler and easy to design for low performance applications whereas active filters have better performance for more demanding applications.

9.1 Decibel Scale

The frequency response of filters often deals with gain and frequency values that vary over many orders of magnitude. Therefore, it is often more convenient to describe these quantities using a logarithmic scale. The conventional scale used to describe gain is the decibel (dB) scale. For a linear gain A, the gain in dB is,

$$A_{dB} = 20\log_{10}(A)$$

Therefore, a gain of 1 is equivalent to 0 dB, a gain of 10 is equivalent to 20 dB, a gain of 100 is equivalent to 40 dB, and so on. Other commonly used dB values include -3 dB which is a gain of approximately $1/\sqrt{2} = 0.707$ and -6 dB which is approximately 0.5. A list of commonly encountered dB values and their linear equivalents is shown in *Table 9.1*. It is important to remember that just as linear gain is a relative measurement of the input and output of a circuit, gain on the dB scale is also a relative measure and is not tied to a specific voltage.

Gain (dB)	Gain (linear)
-60 dB	0.001
-40 dB	0.01
-20 dB	0.1
-6 dB	0.5 (approximate)
-3 dB	0.707 (approximate)
0 dB	1
20 dB	10
40 dB	100
60 dB	1000

Table 9.1: Commonly encountered gain values in dB, with their linear equivalents.

9.2 Single-pole Passive Filters

The simplest filter is a RC low-pass filter shown in *Figure 9.1* The transfer function of this filter is

$$T(s) = \frac{1}{sRC + 1},$$

and the frequency response is shown in Figure ###. This is called a single-pole filter since the denominator of the transfer function has only one root. The pass band gain of the filter is 0 dB or unity, while the stop band attenuation increases by 20 dB per decade. A decade is a 10X increase in frequency. The cutoff frequency is defined as the frequency where the input signal has been attenuated by a factor of 0.707, which is equivalent to a gain of -3 dB. The cutoff frequency of the single-pole filter is

$$\omega_c = \frac{1}{RC}.$$

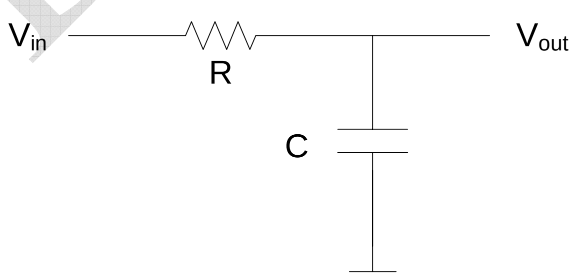

Figure 9.1: Single-pole RC low-pass filter

###Insert frequency response of single-pole RC filter here
Figure ###: Frequency response of a single-pole RC filter

A single-pole high-pass filter can be made by simply switching the position of the resistor and capacitor as shown in Figure ###. The frequency response is shown in *Figure 9.2*.

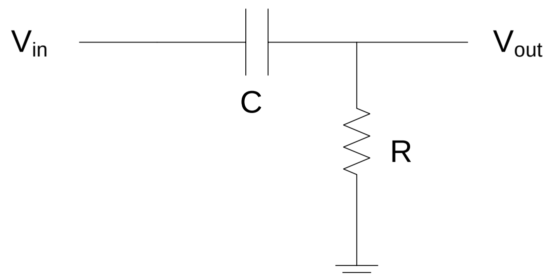

Figure 9.2: Single-pole RC high-pass filter

9.3 Metrics for Filter Design

The frequency response of a filter is specified by its stop-band attenuation above the cutoff frequency. For example, a single-pole filter will attenuate signals in the stop-band by an additional factor of 10 (20dB) for every decade above the cutoff frequency. Therefore, for the low-pass filter in the previous example, at $\omega = 0.1\ \omega_c$, the transfer function is approximately equal to 0.1; at $\omega = 0.01\ \omega_c$, the transfer function is approximately equal to 0.01. If more stop-band attenuation is desired, then a filter with more poles in the transfer function is needed. As shown in Figure ###, a 2-pole filter will attenuate stop-band signals by a factor of 100 (40dB) for every decade above the cutoff frequency; a 3-pole filter will attenuate stop-band signals by a factor of 1000 (60dB) for every decade above the cutoff frequency.

###Insert figure here###
Figure ###: Comparing 1-pole, 2-pole, and 3-pole filter responses.

9.4 Two-pole Passive Filter

The stop-band attenuation of a filter can be using a 2-pole passive filter as shown in *Figure 9.3*.

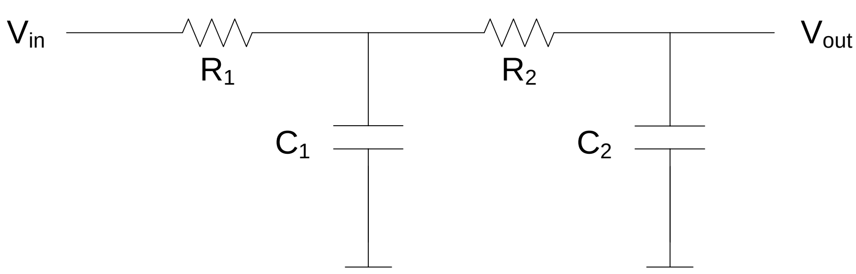

Figure 9.3: Cascaded low-pass filter

From this point on will be a discussion on why passive filter is not cascadable, and motivate the need for active filters.

9.5 Active filters

A major advantage of active filters is that they can be designed to have high input impedance and low output impedance. This means that the signal load can be isolated from the signal source and therefore filter sections can be chained together like LEGO's™ in order to obtain a filter with a higher-order response. As a pleasant consequence, each filter section can provide gain to the desired signal as required to suit the needs of the system.

9.5.1 First order low pass

9.5.2 First order high pass

9.5.3 Second order low pass

9.5.4 Second order high pass

9.5.5 Bandpass

10 Feedback

10.1 Basics of Feedback

Feedback is a fundamental engineering principle used to regulate the response of systems by using the output of a system to affect its input. The interaction between output and input ensures that a stable relationship is maintained even in the presence of disturbances and variations of system characteristics. The application of feedback is ubiquitous in both natural and engineered systems. For example, in physiological systems, feedback is used to regulate breathing and maintain a constant body temperature; in robotics, feedback is used to regulate the position and angle of robotic arms; and in electronic amplifiers, feedback is used to make ideal linear amplifiers with stable gain.

Feedback loops can be either negative or positive. If the feedback loop serves to decrease the effect of the input, then the feedback is said to be negative. Conversely, if the feedback loop serves to increase the effect of the input, then the feedback is said to be positive. In most instances, negative feedback results in a stable output response whereas positive feedback results an unstable or oscillatory response.

The basic feedback loop is shown in *Figure 10.1* where the output of a system, described by the transfer function $A(\omega)$, is multiplied by a transfer function $f(\omega)$ and subtracted from the input before being re-inputed into the system. The transfer function $A(\omega)$ is known as the open-loop response while the overall response of the feedback loop is known as the closed-loop response. The negative sign on the feedback loop is placed there by convention since a majority of engineering feedback systems use negative feedback. Since both A and f are functions of frequency, ω, the overall response will also be a function of ω.

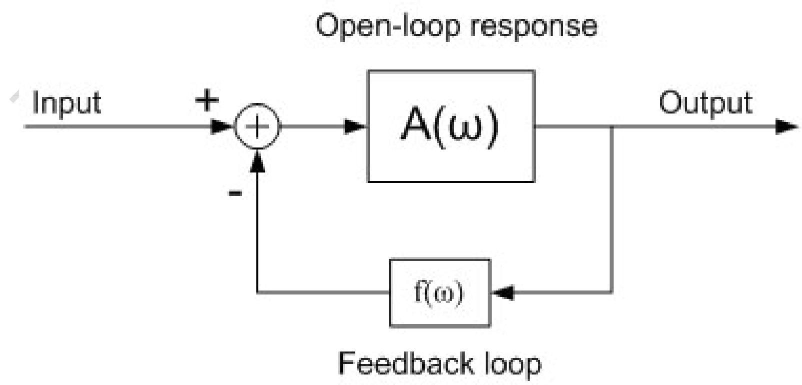

Figure 10.1: Basic configuration of a feedback system

When the feedback loop shown in *Figure 10.1* is applied to an op amp, a number of performance enhancements can be obtained including precise control of the amplifier's gain, improved linearity, increased bandwidth, reduced sensitivity to external disturbances, reduced sensitivity to component variation, and active control of circuit impedances. As with any feedback system, these performance enhancements come at a price: if improperly designed, the system may become unstable and spontaneous oscillations can occur. In fact, the design of feedback systems is often a trade-off between stability and performance, which requires careful analysis by the designer.

10.2 Analysis of Feedback Systems

Feedback systems are analyzed in the frequency domain and provide electronic circuit designers with two important results: the system response and stability. The system response is the closed-loop transfer function of the system which in frequency domain analysis is expressed as a ratio of output to input. The stability of the system measures how likely spontaneous oscillations will occur, which is most commonly expressed by the metrics of phase margin and gain margin.

Feedback analysis uses a block diagram representation of the system based around the path of signal flow. In this representation, the signal can be voltage, current, or any other physical quantity. All physical mechanisms and circuit networks are abstracted as linear operations on the signal, which consists of either multiplying the signal by a constant factor or summing two or more signals together. It is important to remember that these linear operations are complex and that means both the amplitude and phase of the signal may be modified.

10.3 Non-inverting Amplifier

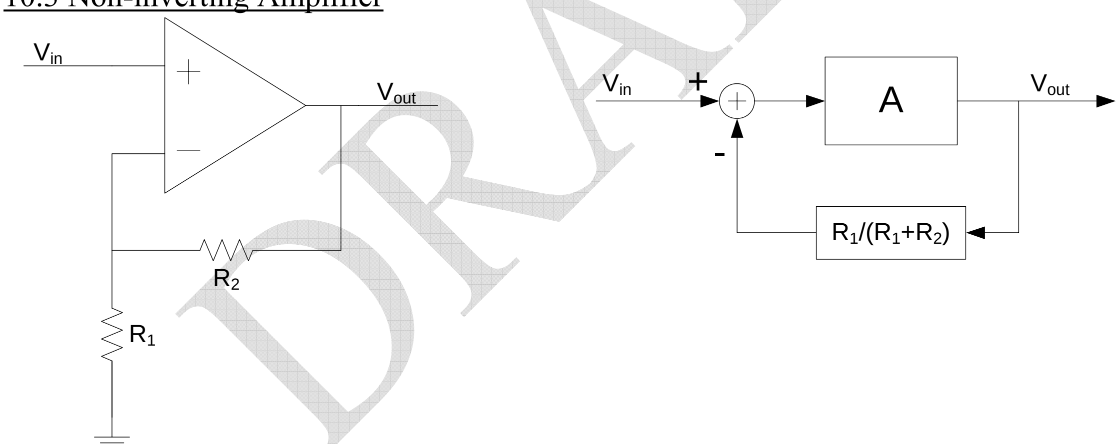

Figure 10.2: Circuit and block diagram representation of an op amp non-inverting amplifier

The block diagram representation of the op amp non-inverting amplifier is shown in *Figure 10.2*. The summation point has both a non-inverting and an inverting input corresponding to the plus and minus terminals of the op amp. The open-loop response of the op amp is modeled as a real multiplicative factor *A*. For ideal op amps, *A* is infinite, while for real op amps, *A* is very large, but finite. The output voltage is attenuated by the voltage divider $R_1/(R_1+R_2)$ and fed back to the minus input of the op amp. The transfer function is calculated as,

$$V_{out} = \mathrm{A}\left(V_{in} - V_{out}\left(\frac{R_1}{R_1+R_2}\right)\right), \text{ which means}$$

$$\frac{V_{out}}{V_{in}} = \frac{\mathrm{A}}{1 + \mathrm{A}\left(\dfrac{R_1}{R_1 + R_2}\right)}.$$

As A becomes very large,

$$\frac{V_{out}}{V_{in}} = 1 + \frac{R_2}{R_1}.$$

This example illustrates a fundamental design principle of op amp circuits: If A is very large, then the closed-loop gain is dependent only on the feedback resistors R_1 and R_2. Therefore, the closed-loop gain can be made very accurate and stable since very accurate and stable resistors can be purchased. **Effectively, op amps use feedback to exchange high open-loop gain for stable closed-loop gain.** Additionally, the only requirement in making this amplifier work as an ideal amplifier is that A must be large. A does not need to a particular value, nor does it need to be linear with input (although, A should maintain the same sign for all possible inputs).

10.4 Inverting Amplifier

The block diagram representation of an inverting amplifier is shown in *Figure 10.3*. Both the input and output can be modeled as independent sources and superposition can be applied to determine the signal applied to the minus terminal of the op amp. Both the input and feedback signals should be negative before the summation point; however, in keeping with the convention, the input side of the summation is positive and the feedback side is negative.

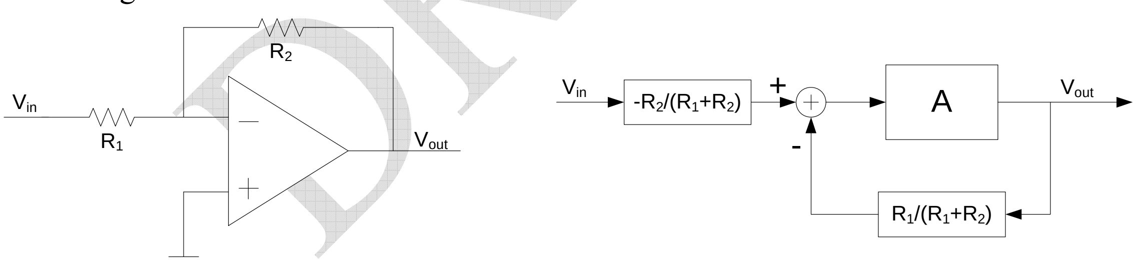

Figure 10.3: Circuit and block diagram representation of an inverting amplifier

The closed-loop transfer function is

$$\frac{V_{out}}{V_{in}} = -\left(\frac{R_2}{R_1 + R_2}\right)\left(\frac{\mathrm{A}}{1 + \mathrm{A}\left(\dfrac{R_1}{R_1 + R_2}\right)}\right).$$

As A becomes very large,

$$\frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}.$$

10.5 Precision Peak Detector

The circuit for a simple peak detector, shown in *Figure 6.6* in section 6.2.1, uses a diode, capacitor, and resistor. This circuit has a problem that the output signal is always one diode drop, 0.7V, below the input signal. Using an op amp and feedback it is possible to make a precise version of this peak detector that automatically compensates for the voltage drop of the diode. The circuit for this peak detector is shown in *Figure 10.4* with a diode connected to a resistor and capacitor, as before. The difference is that now the circuit input goes to the input of an op amp and the output signal from the diode is fed back into the inverting input of the op amp.

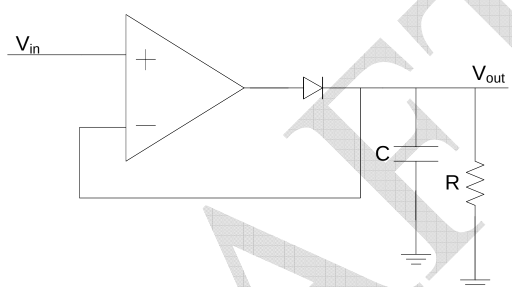

Figure 10.4: Precision peak detector circuit

###The precision peak detector circuit can be analyzed by considering two configurations of this circuit separately: (1) when the diode is forward biased, and (2) when the diode is reversed biased. This technique enables the application of linear analysis while avoiding the inherent nonlinearity of the diode.

The equivalent circuit of the peak detector for the case of a forward biased diode is shown in *Figure 10.5*. The diode is assumed to have zero resistance in the forward direction and has a constant voltage drop of V_D. In the feedback block diagram, the voltage drop can be represented as an additive signal to the output of the op amp. The output voltage of this equivalent circuit is

$$V_{out} = V_{in}\left(\frac{\mathrm{A}}{\mathrm{A}+1}\right) + \frac{V_D}{\mathrm{A}+1}.$$

Since the open-loop gain A for an op amp is typically 10^5-10^6, the output follows the input almost exactly while the diode voltage drop is attenuated by A+1, which is negligibly small compared to the signal amplitude. Therefore, $V_{out} = V_{in}$ when the diode is forward biased.

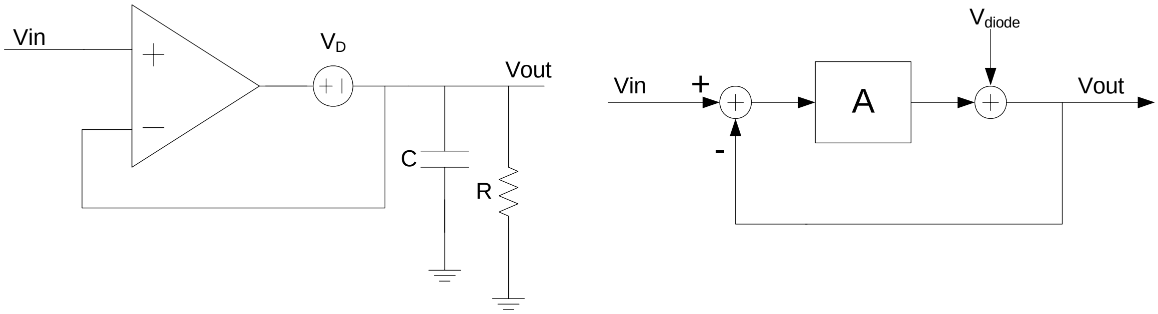

Figure 10.5: Equivalent circuit for the precision peak detector when the diode is forward-biased

The equivalent circuit of the peak detector for the case of reverse-biased diode is shown in *Figure 10.6*. The diode in reverse bias is assumed to behave as a very large resistor with resistance R_D. The open-loop gain of this feedback circuit is A·D, where A is the open-loop gain of the op amp and D is the signal attenuation resulting from R_D with C and R. Specifically,

$$D = \frac{\left(R \parallel \frac{1}{sC}\right)}{\left(R_D + R \parallel \frac{1}{sC}\right)},$$

Since $R_D \approx 10^9\,\Omega$ for reverse biased diodes and $R \approx 10^6\,\Omega$, $\mathrm{D} < 10^{-3}$. This means that the signal transfer from the output of the op amp to V_{out} is attenuated by a factor of 10^3 when the diode is reverse biased. Therefore, V_{out} essentially maintains the voltage on the capacitor C and is not affected by the output of the op amp when the diode is reverse biased ($V_{in} < V_{out}$).

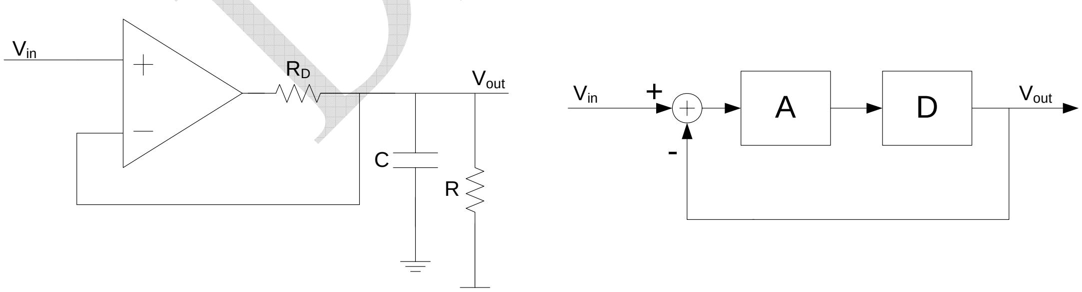

Figure 10.6: Equivalent circuit for the precision peak detector when the diode is reversed biased

<u>10.6 Op amp Frequency Response</u>

Up to this point, it has been assumed that the open-loop gain of the op amp is not only large, but is also constant with frequency. However, the open-loop gain of an op amp begins to decreases at a relatively low frequency, with a single pole roll-off characteristic of

$$A_{OL} = \frac{\mathrm{A}}{1+\tau s}.$$

An example of an op amp open-loop gain versus frequency plot is shown in Figure ###, taken from the OPA340 datasheet. The maximum gain is shown to be 120 dB (10^6), with and the roll-off frequency is 5 Hz.

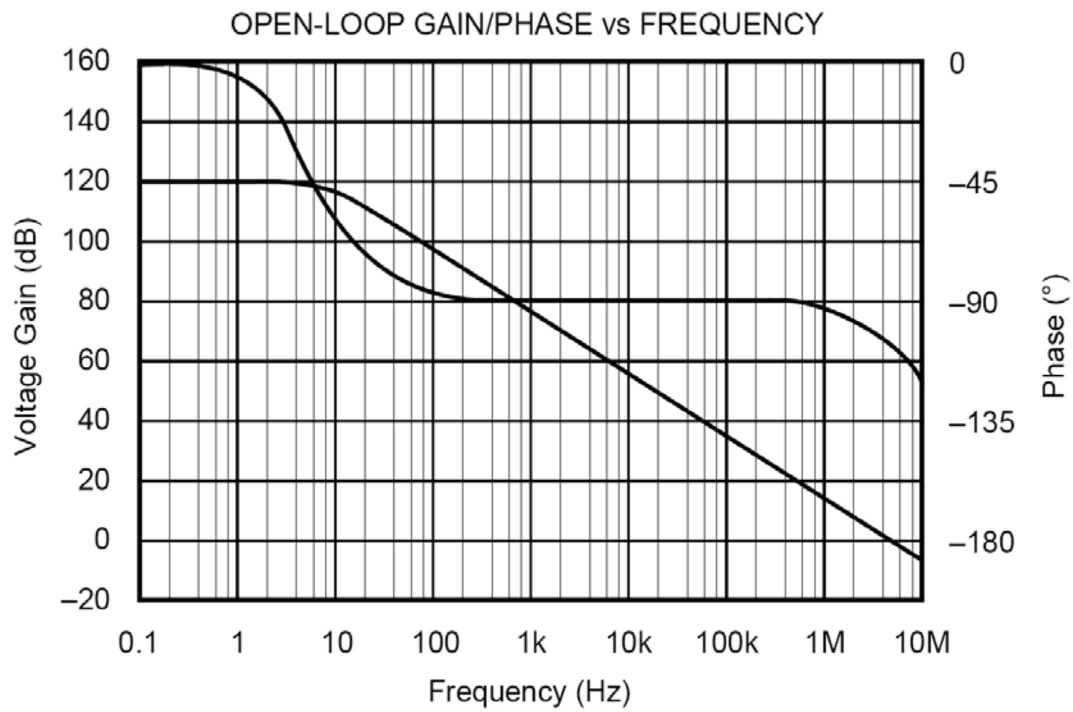

Figure 10.7: An example open-loop gain and phase response of an op amp, taken from the OPA340 datasheet.

The non-inverting amplifier is shown again in Figure ###, having a frequency dependent open-loop gain. The transfer function is

$$\frac{V_{out}}{V_{in}} = \frac{\dfrac{A}{(\tau s+1)}}{1+\left(\dfrac{Af}{\tau s+1}\right)}, \text{ where } f = \frac{R_1}{R_1+R_2}.$$

Therefore,

$$\frac{V_{out}}{V_{in}} = \frac{\frac{1}{f}}{\tau s\left(\frac{1}{\mathrm{A}f}\right) + \left(\frac{1}{\mathrm{A}f}\right) + 1}.$$

Since $Af >> 1$, the $\left(\frac{1}{\mathrm{A}f}\right)$ term can be neglected, and

$$\frac{V_{out}}{V_{in}} \approx \frac{\left(\frac{1}{f}\right)}{\left(\frac{1}{\mathrm{A}f}\right)\tau s + 1}.$$

The closed-loop system has a gain of $\frac{1}{f}$ and a bandwidth of $\frac{\mathrm{A}f}{\tau}$.

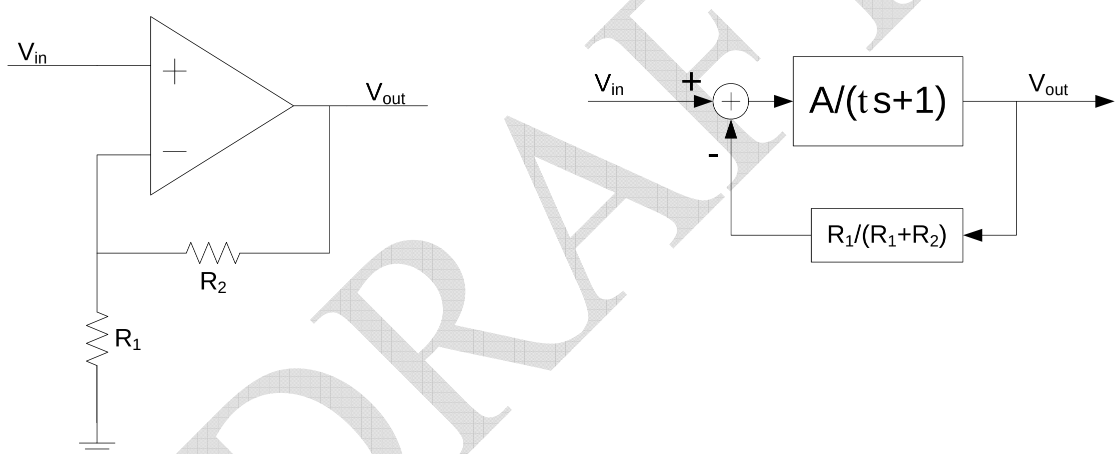

Figure 10.8: Non-inverting amplifier with op amp frequency dependence

10.7 Stability of Feedback Circuits